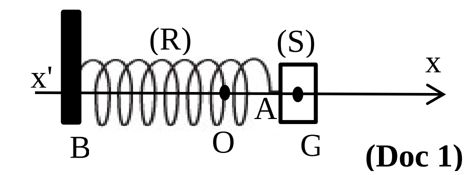
المادة: الفيزياء الشهادة: الثانوية العامّة الفرع: العلوم العامة نموذج رقم 2 المدّة: ثلاث ساعات	الهيئة الأكاديميّة المشتركة قسم: العلوم	المركز التربوي للبحوث والانماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي 2016-2017 وحتى صدور المناهج المطوّرة)

This test includes four mandatory exercises. The use of non-programmable calculators is allowed.

Exercise 1 (6½ points) Oscillations of a horizontal elastic pendulum

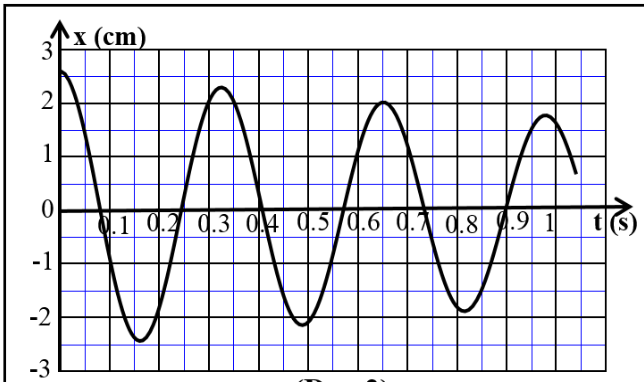


(Doc 1)

An elastic pendulum (R) is formed of a solid (S), of mass m, attached to the extremity A of a horizontal spring of stiffness $k = 80$ N/m; the other extremity B of the spring is attached to a fixed support as shown in the adjacent document (Doc 1).
The center of inertia G of the solid can move along a horizontal axis $x'x$.
At equilibrium, the center of inertia G of (S) is confounded with the origin O of the axis x'x. We shift the solid from its equilibrium position and then we release it from rest at the instant $t_0 = 0$. G starts oscillating on either side of its equilibrium position O.
At an instant t, the abscissa of G is x and the algebraic value of its velocity is $v = \frac{dx}{dt} = x'$.
The horizontal plane passing through G is the reference level of the gravitational potential energy.

1) Free undamped oscillations
We neglect the force of friction.
1-1) Write down, at an instant t, the expression of the mechanical energy of the system (pendulum -Earth).
1-2) Derive the second order differential equation in x that describes the motion of (S).
1-3) Deduce the expression of the proper period T_0 of these oscillations.

2) Free damped oscillations
In reality, the friction force has a certain value. Taking into account the previous initial conditions, a device allows to register the variations of x as a function of time t as shown in the adjacent document (Doc 2).
2-1) Referring to the graph, determine the pseudo-period T of the oscillations.
2-2) Calculate the average power dissipated between the instants $t_0 = 0$ and $t_1 = 3T$.



(Doc 2)

3) Forced oscillations
We connect now the extremity B of the spring to a vibrator of adjustable frequency f_v and of constant amplitude. We give f_v different values and we register, for every value of f_v, the corresponding value of the amplitude x_m of the oscillations of G as shown in the document (Doc 3) below.

(Doc 3)

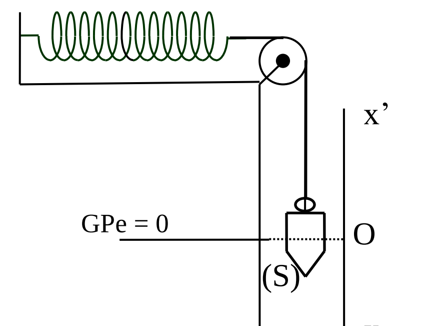
f_v (Hz)	1.5	2	2.5	2.8	3	3.2	3.3	3.6	4	4.5
X_m (cm)	0.4	0.6	1	1.5	2.1	2.3	2	1.5	1	0.7

3-1) Referring to the table, determine the approximate value of the proper period of the oscillations of (R).
3-2) Determine the approximate value of m.
3-3) Sketch the graph giving the variation of x_m as a function of f_v.
3-4) Trace, with justification, the shape of the previous curve when the force of friction has a greater value.

Exercise 2 (7½ points) **Synchronous pendulums**

1) Elastic pendulum

A spring, of force constant k and of negligible mass, is placed on a smooth horizontal table. The left end of the spring is fixed to a support and the right end is connected to the end of a massless string passing over a light pulley as shown in the adjacent document (Doc 4). A particle (S), of mass m, is tied to the other end of the string. At equilibrium, (S) is at O. Take the horizontal plane passing through O as the reference level of the gravitational potential energy and g = 10 m/s^2. Neglect all resistive forces.



(Doc 4)

1-1) When (S) is at equilibrium, it coincides with the origin O of the vertical axis x'ox, and the spring is extended by $\Delta\ell$.
Show that $\Delta\ell = \frac{mg}{k}$.

1-2) The particle, pulled down by 4 cm, is released from rest at the instant $t_0 = 0$. At an instant t, the abscissa of the particle is x and the algebraic value of its velocity is $v = \frac{dx}{dt} = x'$.

1-2-1) Show that, at an instant t, the expression of the mechanical energy of the system [(S), Earth, spring, string, pulley] is given by: $ME = \frac{1}{2}k(\Delta\ell + x)^2 - mgx + \frac{1}{2}mv^2$.

1-2-2) Determine the second order differential equation, in x, that describes the motion of (S).

1-2-3) Deduce the expression of the proper angular frequency ω_0 of the pendulum and give that of its proper period T_0 in terms of $\Delta\ell$ and g.

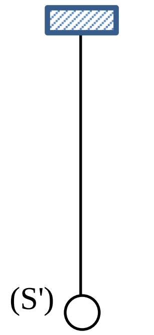
1-2-4) Determine the time equation of the motion of (S) knowing that it is of the form: $x = x_m\sin(\omega_0 t+\varphi)$.

2) Simple pendulum

A simple pendulum is formed of an inextensible and massless string of length L and a particle (S') of mass m as shown in the adjacent document (Doc 5). Suspended in a proper way, (S') is shifted from its equilibrium position by an angular abscissa $\theta_0 = 0.10$ rd, and then released from rest at the instant $t_0 = 0$. The pendulum performs oscillations of angular amplitude $\theta_m = 0.10$ rd. At an instant t, the angular abscissa of the pendulum is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

Take the horizontal plane passing through the equilibrium position of (S') at as the reference level of the gravitational potential energy and g = 10 m/s^2. Neglect all resistive forces.

Take whenever needed, for small values of θ, (θ in rd): $\cos\theta = 1 - \frac{\theta^2}{2}$ or $\sin\theta = \theta$.



(Doc 5)

2-1) Determine, at an instant t, the expression of the mechanical energy of the system (pendulum-Earth).

2-2) Determine the second order differential equation, in θ, that describes the motion of the pendulum.

2-3) Deduce the expression of the proper angular frequency ω'_0 of this pendulum and give that of its proper period T'_0 in terms of L and g.

2-4) Determine the time equation of the motion of the pendulum knowing that it is of the form: $\theta = \theta_m\sin(\omega'_0 t + \varphi')$.

3) Comparison

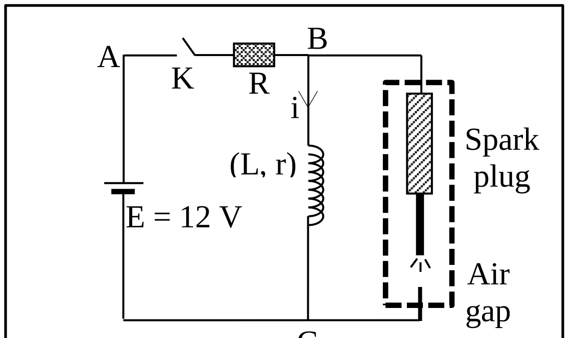
Compare the proper periods T_0 and T'_0 of these pendulums and give the condition to be satisfied by an elastic pendulum and a simple pendulum to be synchronous.

Exercise 3 (6½ points) Sparks in a Car ignition system

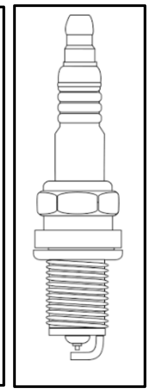
The ability of a coil, to oppose rapid changes in current, makes it very useful for spark generation.
The engine of a car requires that the fuel-air mixture in each cylinder must be ignited at proper times. This is achieved by means of a spark plug, which essentially consists of a pair of electrodes separated, at a specific distance, by an air gap. By creating a large voltage (a few tens of thousands of volts) between the electrodes, a spark is formed across the gap, thereby igniting the fuel.
The coil of a car ignition system has an inductance L = 20 mH and a resistance r = 2 Ω.
The electromotive force of car battery is: E = 12 V.

1) Switch K is closed

The adjacent document (Doc 6) shows the circuit of a spark plug in a car where the resistor used for protection is of resistance R= 4Ω.
At the instant $t_0 = 0$, the switch K of the circuit is closed.



(Doc 6)



1-1) The current in the branch of the spark plug is considered zero. Justify.
1-2) At an instant t, the circuit carries the current i. Using the law of addition of voltages, determine the differential equation in i.
1-3) Deduce, in steady state, the current I_0 carried by the circuit.
1-4) Calculate the time needed by the current i to reach, practically, its maximum value I_0 knowing that the time constant τ of the circuit is: $\tau = \frac{L}{(R+r)}$.
1-5) Calculate the maximum magnetic energy stored by the coil.
1-6) Determine, in steady state, the voltage across the air gap of the spark plug.
1-7) A spark is a visible disruptive discharge of electricity between two electrodes of high voltage. It is preceded by an ionization of the gas (air - fuel) then followed by a rapid heating effect that burns the fuel. Specify if sparks are created in the air gap during the growth of the current in the circuit.

2) Switch K is opened

2-1) When the switch K is opened, the current drops to zero during 1 μs. Determine the voltage developed across the electrodes of the plug.
2-2) Specify if sparks are produced in the air gap.
2-3) The sparks in the air gap get weaker as the distance between the electrodes gets larger. Explain why the spark plug must be changed after being used for a long time.

Exercise 4 (7 points) Nuclear reactions

Consider the following four reactions (1), (2), (3) and (4) and the masses of some nuclei.

	$^{1}_{1}H$	$^{2}_{1}H$	$^{3}_{1}H$	$^{4}_{2}He$	$^{1}_{0}n$	$^{235}_{92}U$	$^{140}_{54}Xe$	$^{94}_{38}Sr$
m(u)	1.0073	2.0141	3.0155	4.0015	1.0087	235.0439	139.9216	93.9153

1 u = 1.66×10^{-27} kg = 931.5 MeV/c^2; c = 3×10^{8} m/s; 1 eV=1.6×10^{-19} J.

$^{235}_{92}U \rightarrow {}^{231}_{90}Th + {}^{A}_{Z}X$ (1)

$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H$ (2)

$^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$ (3)

$^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2({}^{1}_{0}n)$ (4)

1) Give the type of each of these four reactions.

2) Consider the reaction (1).

- **2-1)** Calculate A and Z indicating the laws used.
- **2-2)** Name the particle X and give its symbol.

3) The deuterium nuclei undergo the nuclear reactions (2) and (3).

- **3-1)** Write the overall reaction (5) that takes place.
- **3-2)** Determine, in MeV and in joule, the energy liberated by this reaction.

4) Consider the reaction (4).

- **4-1)** Calculate, in u and in kg, the mass lost.
- **4-2)** Determine the released energy.
- **4-3)** The released energy by the atomic bomb dropped at Hiroshima was estimated to be the equivalent to 15 kilotons of dynamite or 63×10^{12} J.
  - **4-3-1)** Calculate the number of uranium-235 nuclei that underwent fission in this bomb assuming that all of the fission reactions took place as reaction (4) from the energetic point of view.
  - **4-3-2)** Determine the mass of reacting uranium-235 necessary to release this energy.

5) The combustion of a mass m_1 = 1 kg of fuel oil liberates an amount of energy E = 4.3×10^{7} J.

- **5-1)** Determine the mass m_2 of the deuterium nuclei and the mass m_3 of uranium nuclei that may produce this energy.
- **5-2)** Classify in ascending order the masses m_1, m_2 and m_3 and indicate the substance that is preferable to be used to obtain energy, regardless of other factors.

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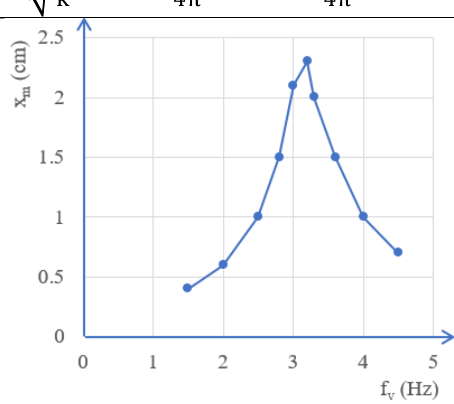
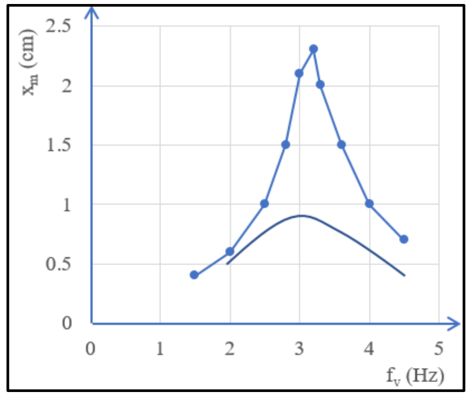
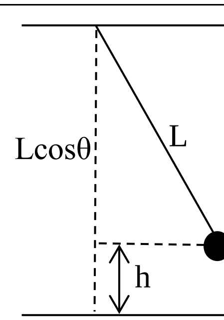
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي 2016-2017 وحتى صدور المناهج المطوّرة)

Exercise 1 (6½ points) Oscillations of a horizontal elastic pendulum

Question	Answer	Mark
1-1	ME = KE + PE $\forall t$ $ME = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$	¼ ¼
1-2	There is no friction, so the mechanical energy of the system is conserved Then ME = constant $\forall t$ $\frac{dME}{dt} = mvv' + kxx' = 0 \quad \forall t$ $mx'\left(x'' + \frac{k}{m}x\right) = 0 \quad \forall t$ The product of two physical quantities is always nil, but mx' is not always nil, we get: $x'' + \frac{k}{m}x = 0$	½ ½
1-3	The differential equation is of the form $x'' + \omega_0^2 x = 0$ The oscillator undergoes simple harmonic oscillation of proper angular frequency $\omega_0 = \sqrt{\frac{k}{m}}$ the proper period is thus: $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$	¼ ¼
2-1	Using the graph, 3 T = 0.98 then T= 0.326 s	½
2-2	for $t_0 = 0$, x_0 =2.6 cm, $ME_0 = \frac{1}{2} kx_0^2 = 0.02704$ J ($KE_0 = 0$ because the elongation is maximum); for t = 3T, x = 1.8 cm, $ME = \frac{1}{2} kx^2 = 0.01296$ J (KE = 0 because the elongation is maximum); $P_{av\ diss} = \frac{\text{Energy lost}}{\text{duration}} = \frac{0.02704-0.01296}{0.98} = 0.0144$ W	½ ½
3-1	Referring to the graph, damping is relatively very weak. In this case, the resonant frequency is too close to the proper frequency of the pendulum. f_0 corresponds to the highest amplitude, f_0 = 3.2 Hz, thus, $T_0 = 1/f_0 = 0.3125$ s	½ ½
3-2	$T_0 = 2\pi\sqrt{\frac{m}{k}}$; $m = \frac{T_0^2 \times k}{4\pi^2} = \frac{(0.3125)^2 \times 80}{4\pi^2} = 0.198$ kg	½ ½
3-3		½

3-4	When the force of friction increases, the maximum value of the amplitude of the curve of resonance becomes smaller (the bandwidth larger and the resonance frequency smaller). When the force of friction becomes greater, the phenomenon of resonance disappears, and (S) is then sensitive to a large band of frequencies, the bandwidth becomes larger. *Remark: The shape of the curve must be in accordance with the initial conditions and in respect for the problem situation.*  	½

Exercise 2 (7½ points) Synchronous pendulums

Question	Answer	Mark
1-1	The Particle S is at equilibrium: W = T $mg = k\Delta\ell \Rightarrow \Delta\ell = \frac{mg}{k}$	½ ½
1-2-1	$ME = KE + PE_g + PE_e = ½\, k(\Delta\ell + x)^2 - mgx + ½\, mv^2$	½
1-2-2	There is no friction thus ME is conserved $ME = ½\, k\Delta\ell^2 + ½\, kx^2 + kx\Delta\ell - mgx + ½\, mv^2 = \text{constant} \quad \forall t\ (k\,\Delta\ell = mg)$ $\frac{dME}{dt} = 0 + kxx' + mvv' = 0 \quad \forall\, t \Rightarrow mx'\left(x'' + \frac{k}{m}x\right) = 0 \quad \forall\, t$ The product of two physical quantities is always nil, but mx' is not always nil, we get: $x'' + \frac{k}{m}x = 0$	½ ½
1-2-3	The differential equation is of the form $x'' + \omega_0^2 x = 0$ The oscillator undergoes simple harmonic oscillations of proper angular frequency $\omega_0 = \sqrt{\frac{k}{m}}$ the proper period is thus: $T_0 = 2\pi\sqrt{\frac{m}{k}} T_0 = 2\pi\sqrt{\frac{\Delta\ell}{g}}$	¼ ¼
1-2-4	$\omega_0 = \sqrt{\frac{k}{m}}$ $x = x_m\sin(\omega_0 t + \varphi)$; $v = x_m\omega_0\cos(\omega_0 t + \varphi)$; at $t_0 = 0$: $x_0 = x_m\sin(\varphi) > 0$ and $v_0 = x_m\omega_0\cos(\varphi) = 0$ then $\varphi = \frac{\pi}{2}$ rd $x_0 = x_m = 4$ cm. $x = 4\sin(\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2})$; (t in s and x in cm)	¼ ¼ ¼ ¼
2-1	$ME = PE_g + KE$ $ME = mgh + ½\, I\,\theta'^2 = mgL(1 - \cos\theta) + ½\, I\theta'^2$ Knowing that $\theta = 0.1$ rd < 6°, $\cos\theta = 1 - \frac{\theta^2}{2}$ $ME = ½\, mgL\theta^2 + ½\, mL^2\,\theta'^2$  	¼ ¼ ½

2-2	$ME = ½\, mgL\theta^2 + ½\, mL^2\, \theta'^2 = constant \quad \forall t$ $\frac{dME}{dt} = 0$ then: $mgL\theta\theta' + mL^2\, \theta'\theta'' = 0$; $mL\theta'$ is not always nil $g\theta + L\theta'' = 0 \Rightarrow \theta'' + (g/L)\,\theta = 0$	¼ ½
2-3	The differential equation has the form to $\theta'' + \omega'^2_0\theta = 0$ The oscillator undergoes simple harmonic oscillation of proper angular frequency ω'_0 with $\omega'^2_0 = \frac{g}{L}$, thus, $\omega'_0 = \sqrt{\frac{g}{L}}$ The proper period is thus: $T'_0 = \frac{2\pi}{\omega'_0} = 2\pi\sqrt{\frac{L}{g}}$	½
2-4	$\theta = \theta_m \sin(\omega'_0 t + \varphi')$; $\theta' = \theta'_m\omega'_0\cos(\omega'_0 t + \varphi')$; at $t_0 = 0$: $\theta_0 = \theta_m\sin(\varphi') > 0$ and $\theta'_0 = \theta_m\omega'_0\cos(\varphi') = 0$ then $\varphi' = \frac{\pi}{2}$ rd $\theta_0 = \theta_m = 0{,}1$ rd $\theta = 0.1\sin\left(\sqrt{\frac{g}{L}}\, t + \frac{\pi}{2}\right)$; (t in s and θ in rd)	¼ ¼ ¼
3	$T_0 = 2\pi\sqrt{\frac{\Delta\ell}{g}}$ and $T'_0 = 2\pi\sqrt{\frac{L}{g}}$ The two pendulums are synchronous, $T_0 = T'_0 \Rightarrow \Delta\ell = L$	½

Exercise 3 (6½ points) **Sparks in a Car ignition system**

Question	Answer	Mark
1-1	Since the branch of the spark plug is an open circuit due to the air gap	½
1-2	$u_{AC} = u_{AB} + u_{BC}$; $E = Ri + (ri + L\, di/dt)$ $\frac{E}{L} = \frac{(R+r)}{L}\, i + \frac{di}{dt}$	½ ½
1-3	In steady state: $i = I_0$ = constant $\frac{E}{L} = \frac{(R+r)}{L} I_0 + \frac{di}{dt}$, $(di/dt = 0)$ $E = (R+r)\, I_0 \Rightarrow I_0 = E/(R+r) = 12/6 = 2$ A	½ ½
1-4	$\Delta t = 5\tau = 5\frac{L}{(R+r)} = 5\times 0.02/6 = 0.0167$ s	½
1-5	$E_{mag}(max) = ½\, LI_0^2 = ½\times 0.02\times 2^2 = 0.04$ J	½
1-6	$u_{air\ gap} = u_{BC} = rI_0 = 2\times 2 = 4$ V	½
1-7	No since $u_{air\ gap} = 4$V is small.	½
2-1	The average induced e.m.f. is: $e_{av} = -L(\Delta i/\Delta t) = -0.02\times(0-2)/10^{-6} = 40000$ V $\Rightarrow \lvert u_{airgap}\rvert = u_{CB} \approx 40000$ V (since ri is assumed very small).	½ ½
2-2	Yes, sparks are produced in the gap because the voltage is very high.	½
2-3	The sparks produced in the air gap will melt gradually the electrodes of the spark plug; this causes an increase in the distance between them and consequently the sparks get fainter and then the plug must be replaced by a new one.	½

Exercise 4 (7 points) Nuclear reactions

Question	Answer	Mark
1	(1): Natural radioactivity (2): and (3): fusion (4): fission	¼ ½ ¼
2-1	By applying Soddy's laws: Conservation of the mass number: 235 = 231 + A $\Rightarrow$ A = 4 Conservation of the charge number: 92 = 90 + Z $\Rightarrow$ Z = 2	¼ ¼ ¼
2-2	This particle is the helium-4 nucleus; its symbol is $^{4}_{2}He$.	¼
3-1	Adding 2 and 3 we obtain: $3\,^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{3}_{1}H + ^{1}_{1}H + ^{1}_{0}n$ $3\,^{2}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{0}n$ (5)	½
3-2	$E_{Lib.} = \Delta m.c^2 = [(3\times2.0141) - (4.0015 + 1.0073 + 1.0087)] \times 931.5$ $E_{Lib} = 0.0248 \times 931.5 = 23.1012$ MeV $= 3.696\times10^{-12}$ J	½
4-1	$\Delta m = [(1.0087 + 235.0439) - (139.9216 + 93.9153 + 2(1.0087))]$ $\Delta m = 0.1983$ u $= 0.1983\times1.66\times10^{-27} = 3.292\times10^{-28}$ kg	½
4-2	$E_{Lib.} = \Delta m.c^2 = 0.1983\times931.5 = 184.72$ MeV $= 184.72 \times 1.66\times10^{-13}$ J $E_{Lib.} = 29.56\times10^{-12}$ J	½
4-3-1	1 nucleus liberates 29.56×10^{-12} J N nuclei liberate 63×10^{12} J $N = 2.131\times10^{24}$ nuclei	½
4-3-2	The mass lost of 3.292×10^{-28} kg corresponds to 1 nucleus The mass lost of Δm_{total} corresponds to 2.131×10^{24} nuclei Then $\Delta m_{total} = 0.0007$ kg The mass lost of 3.292×10^{-28} kg corresponds to 1 nucleus of mass 235.0439 u The mass lost of 0.0007 kg corresponds to N nuclei of mass m_t Then the mass of the used uranium is: $m_t = 4.99 \times 10^{26}$ u = 0.83 kg.	½ ½
5-1	For the deuterium nuclei: $3m(^{2}_{1}H) = 6.0423$ u $= 10.03 \times 10^{-27}$ kg gives 36.96×10^{-13}J m_2 gives 4.3×10^{7}J Then we need $m_2 = 4.3\times10^{7} \times 10.03 \times 10^{-27}/36.96\times10^{-13} = 1.1669\times10^{-7}$ kg For the uranium nuclei: $m_U = 235.0439$ u $= 3.9\times10^{-25}$ kg gives 29.56×10^{-12} J m_3 gives 4.3×10^{7} J Then we need $m_3 = 4.3\times10^{7} \times 3.9 \times 10^{-25}/29.56\times10^{-12} = 5.67\times10^{-7}$ kg	½ ½
5-2	To liberate the same quantity of energy, 4.3×10^{7} J, we need: Fuel: $m_1 = 1$ kg Deuterium nuclei: $m_2 = 1.1669\times10^{-7}$ kg Uranium nuclei: $m_3 = 5.67\times10^{-7}$ kg $m_2 < m_3 < m_1$ We prefer using deuterium nuclei.	½