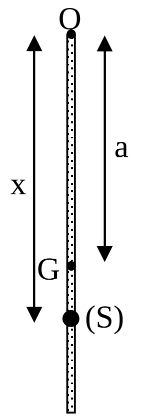
مسابقة في مادة الفيزياء
المدة: ثلاث ساعات

الاسم:
الرقم:

This exam is formed of 4 exercises in 4 pages
The use of a non-programmable calculator is recommended

First exercise : (7 pts)

Mechanical oscillations



The object of this exercise is to study the response of a simple pendulum to the excitations produced by a compound pendulum of adjustable period.
In order to do that, we consider a simple pendulum (R) and a compound pendulum (E). (R) is equiped with a plate of negligible mass, allowing to control the damping due to air. (E) is formed of a homogeneous rod of mass M, of length $\ell = 1$ m and of negligible cross-section, along which a solid (S), taken as a particle of mass M'=M, may slide (E) may oscillate around a horizontal axis (Δ) perpendicular to the rod through its upper extremity O (Figure).
G is the center of gravity of the compound pendulum thus formed and I the moment of inertia of this pendulum about (Δ). Take OG = a and denote by x the distance between the position of (S) and O.
Take : $g = 10$ m/s^2 ; $\pi^2 = 10$; $\sin\theta \approx \theta$ rad pour $\theta \leq 10^0$.

A- Theoretical study

The pendulum (E) is shifted from its equilibrium position by a small angle and then released from resta t the instant $t_0 = 0$. (E) starts to oscillate around its position of stable equilibrum. We neglect all the forces of friction.
At any instant t, OG makes with the vertical through O an angle θ and (E) acquires an angular velocity θ'.
The horizontal plane through O is taken as a gravitational potential energy reference for the system [(E), earth]

1- Show that the expression of the mechanical energy of the system [(E)-earth] may be written as :
$M.E = \frac{1}{2} I\theta'^2 - 2Mga\cos\theta$.

2- Etermine the second order differential equation in θ that governs the motion of (E) for small oscillations ($\theta \leq 10^o$).

3- **a**) Show that the expression of a may be written as : $a = \frac{\ell + 2x}{4}$.

b) The moment of inertia of the rod alone about the axis (Δ) is : $I_1 = M\frac{\ell^2}{3}$.

Show that the expression of the moment of inertia I is given by : $I = \frac{M(\ell^2 + 3x^2)}{3}$.

4- Show that the expression of the proper period T of (E), in terms of x, may be given in the form
$T = \sqrt{\frac{8(1+3x^2)}{3(1+2x)}}$.

B- Experimental study

1- Consider the pendulum (R) alone. It is shifted from its equilibrium position by a small angle, and is then released from rest. The duration t_1 of 10 oscillations is measured and found to be t_1= 16,6 s. Calculate the duration T' of one oscillation.

2- (E) and (R), initially at rest, are coupled by means of a spring. For each value of x, the pendulum (E) is shifted from its equilibrium position by a small angle then released from rest; it causes then (R) to oscillate. We assume that (E) oscillates with a period equal to its proper period T.
When x is made to vary, we notice that the amplitude θ_m of the oscilliations of (R) varies.

a) The oscillations of (R) are said to be forced. Compare then, for each value of x, the period of oscilliations of (R) and that of (E).

b) **i)** We give x the value 0.3 m. In steady state, (R) oscillates with a period T_1 and of amplitude θ_{m1}

ii) We give x the value 0.65m. In steady state, (R) performs oscilliations of period $T_2 = 1,62$ s and of amplitude θ_{m2}. Compare, with justification, θ_{m1} and θ_{m2}.

c) For a certain value of x, and in steady state, (R) oscillates with an amplitude $\theta_{m(max)}$.

i) Give the name of the phenomenon that took place.

ii) Determine the value of x.

d) Trace the shape of the graph that shows the variation of the amplitude θ_m of the oscilliations of (R) as a function of the period T of (E).

e) The plate of (R) is put in such a way so as to increase the friction with air. Trace, on the same sustem of axes of question (d), the shape of the curve that shows the variation of the amplitude θ_m of the oscilliations of (R) as a function of the period T of (E).

<u>Second exercise</u> : (7 pts)

Ignition system in a car

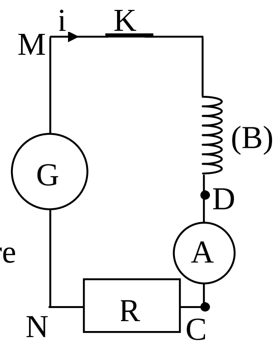


Figure 1

The study of the ignition system in certain cars is reduced to the study of a circuit formed of a coil (B) of inductance L and resistance r, a resistor of resistance R, an ammeter (A) and a switch K, all connected in series across a generator (G) that provides across its terminals M and N a voltage $u_{MN} = E = 12$ V (Figure1).

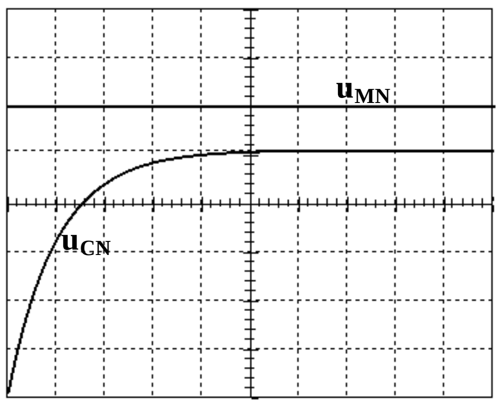
We close the switch K at the instant $t_0 = 0$.

At the instant t, the circuit carries a current i. We display, using an oscilloscope, the voltage u_{MN} on the channel Y_1 and the voltage u_{CN} on the channel Y_2. The waveforms are represented in figure 2.

The vertical sensitivity on both channels is : 2 V / div.

The horizontal sensitivity(time base) is 1 ms /div.

In steady state, the ammeter reads $I_0 = 0,2$ A.



0 Volt **Figure 2**

1- Redraw figure 1 showing on it the connections of the oscilloscope.

2- **a)** Derive the diffenrential equation that governs the variation of the current I as a function of time.

b) **i)** Show that in steady state: $E = (R + r) I_0$ and that $u_{MD} = rI_0$.

ii) Determine R and r using the waveforms and the preceding results.

3- **a)** **i)** Show, using the preceding differential equation, that the voltage u_{CN} satisfies the relation $\frac{RE}{L} = \frac{du_{CN}}{dt} + \frac{R+r}{L}u_{CN}$

ii) Deduce the expression of $\frac{du_{CN}}{dt}$, in terms of R, E et L, at the instant $t_0 = 0$.

iii) The time constant τ is the abscissa of the intersection of the tangent at the origin to the curve u_{CN} and the asymptote to that curve. Show that the expression of τ is : $\tau = \frac{L}{R+r}$.

b) Show, using one of the waveforms, that the value of τ is 1 ms.

c) Deduce the value of L.

4- Determine the maximum energy stored in the coil (B).

5- The above circuit (ignition system) helps, through an intermediary switch, to feed the spark plugs of the car at well determined instants, with the energy needed to make the engine function normally.

The expression of the current I, in the circuit, is given by : $i = I_0(1 - e^{\frac{-t}{\tau}})$.

We define the « rate of storage » of the coil as the ratio of the energy stored in the coil at a given instant to the maximum energy it can store. Determine the minimum duration of closure of the switch so that the rate of storage of the coil is not less than 90.3 %.

Third exercise : (7 pts)

Electromagnetic oscillations

An oscillating circuit is formed of a capacitor of capacitance C =1μF and a coil of inductance L and resistance r. In order to determine L and r, we connect up the circuit whose diagram is represented in figure1. The connections of the oscilloscope are as indicated on this figure.

The E.M.F of the generator is : E = 10 V.

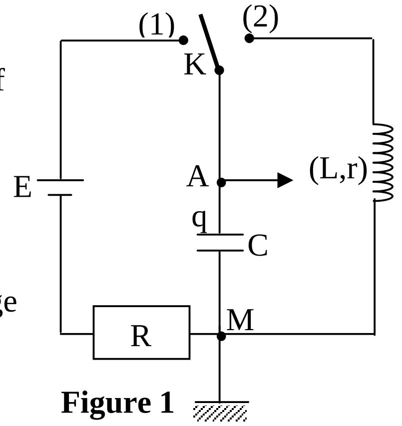


Figure 1

A- Charging the capacitor

The switch K is in position (1). The capacitor is totally charged and the voltage across its terminals is $u_{AM} = U_0$.

1- Determine the value of U_0.

2- Calculate the electric energy stored in the capacitor.

B – Electromagnetic oscillations

The capacitor being totally charged, we move thw switch K to the position (2) at the instant $t_0 = 0$. At the instant t, the circuit carries a current i and the armature (A) carries the charge q.

I – Ideal circuit

In the ideal circuit, we neglect the resistance r of the coil.

1- Redraw figure 1 indicating an arbitrary direction of the current.

2- Derive the differential equation that governs the variation of the voltage $u_{AM} = u_C$ across the terminals of the capacitor as a function of time.

3- Deduce, then, the expression of the proper period T_o of the electric oscilliations in terms of L and C.

4- Give the shape of the curve representing the variation of u_C as a function of time.

5- Specify the mode of electric oscilliations that is taking place in the circuit.

II – Real circuit

The variation of the voltage u_C observed on the screen of the oscilloscope is represented in the waveform of figure2.

1- Specify the mode of electric oscilliations that takes place in the circuit.

2- By referring to the waveform :

a) Give the value of the pseudo-period T of the electric oscillations.

b) Verify that the ratio of two positive extreme values of the voltage u_C is practically equal to a constant a (this is limited to the first four extreme values).

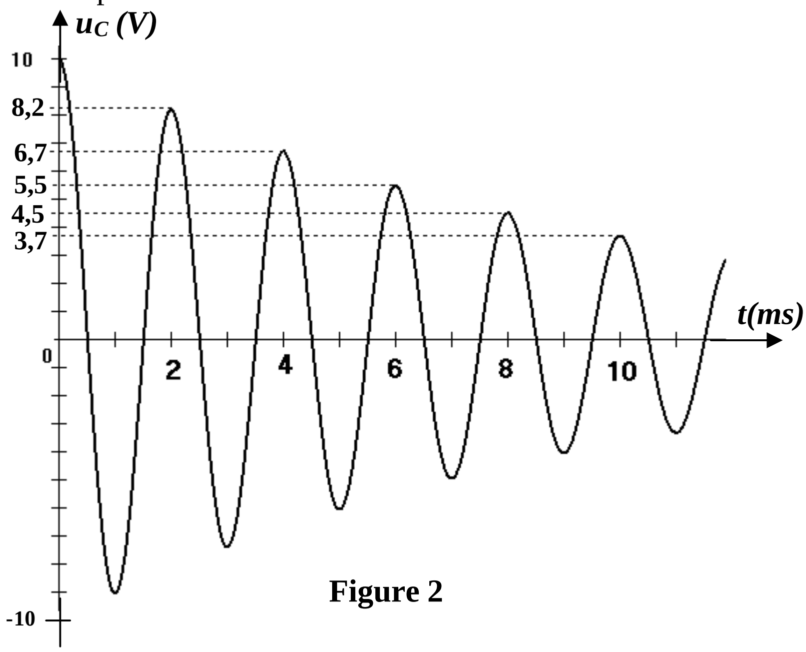


Figure 2

3- We denote by E_n and $E_{(n+1)}$ the electromagnetic energy of the electric oscillator at the instants nT and (n+1)T respectively (n is a positive whole number).

a) The energy stored in the circuit at the instant when the voltage u_C is maximum is electric. Why?

b) Derive the expression of the ratio $\frac{E_{(n+1)}}{E_n}$ in terms of a.

c) Determine L and r knowing that $\frac{E_{(n+1)}}{E_n} = e^{\frac{-r}{L}T}$ and that the expression of the pseudo-period T is:

T est : $\frac{4\pi^2}{T^2} = \frac{4\pi^2}{T_o^{\ 2}} - \frac{1}{4}(\frac{r}{L})^2$.

<u>Fourth exercise</u> : (6 ½ pts)

Radioactivity

The object of this exercise is to show evidence of some characteristics of a thorium nucleus 230 and its role in dating.

<u>Given :</u> speed of light in vacuum : c = 3 $\times 10^8$ ms^{-1}; 1eV = 1,6$\times 10^{-19}$ J;
Avogadro's number : N = 6,02$\times 10^{23}$ mol^{-1};
Planck's cpnstant :h = 6,63$\times 10^{-34}$ J.s; 1u = 931,5 MeV/c^2 ;
masses of the nuclei : m$\left({}^{A}_{88}Ra\right)$ = 225,9770 u; m (${}^{230}_{Z}Th$) = 229,9836 u; m(α) = 4,0015u.

A- Decay of a thorium nucleus 230

The thorium nucleus (${}^{230}_{Z}Th$) is radioactive and is an α emitter . The daughter nucleus is the isotope of the radium$\left({}^{A}_{88}Ra\right)$.

1- **a)** Write the equation of this decay and determine the values of A and Z.
b) Determine the energy liberated by the decay of a thorium nucleus 230.

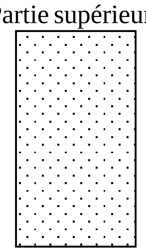
2- A decay of a thorium nucleus 230 at rest, ttakes place without the emission of γ radiation. The daughter nucleus (${}^{230}_{z}th$) obtained has a speed almost zero. Determine the value of the kinetic energy $K.E_1$ of the emmitted α particle.

3- Another decay of a thorium nucleus 230 is accompanied with the emission of a γ radiation of wavelength $6\times 1^{0-12}$ m in vacuum.
a) Calculate the energy of this radiation.
b) Deduce the value of the kinetic energy $K.E_2$ of the emitted α particle.

4- A sample of 1g of thorium nucleus 230of activity A_0= 7,2$\times 10^8$ decays/s is placed near a sheet of aluminum at the instant t_0 = 0. The α particles are stopped by the aluminum sheet whereas the photons are not absorbed.
a) Determine, in J, the energy W transfered to the aluminum sheet during the first second knowing that 50% of the decays are accompanied with γ emission, and that the activity A_o remains practically constant within this second.
b) Calculate the number of nuclei present in 1g of thorium 230. Deduce, in year^{-1}, the value of the radioactive constant λ of thorium 230.

B- Dating of marine sediments

Due to the phenomenon of erosion, a part of the rocks is driven into the oceans.
Some of these rocks contain the radioactive uranium 234 (${}^{234}_{92}U$) which gives thorium 230.
The uranium 234 is soluble in sea-water, whereas thorium is not, but it is accumulated at the bottom of the oceans with other sediments.
We take a specimen, formed of a cylinder from the bottom of the ocean.
This specimen has an upper layer that is just formed, and a lower layer that is formed a time t ago. We take a sample (a) from the upper part and another sample (b), of the same mass, from the lower part.
It is found that the sample (a) produces 720 decays/s and (b) 86.4 decays/s.
Determine t in years.



Solution

<u>First exercice</u> : (7 pts)

A- 1- ME = KE + PE_g ; KE = $\frac{1}{2} I \theta'^2$; PE_g = -2Mgh_G where $h_G = a\cos\theta$ **(3/4 pt)**

2- $\frac{dME}{dt} = 0$ = $I\theta'\theta'' + 2Mga\theta'\sin\theta$; for $\theta \leq 10^o$ we get : $\theta'' + \frac{2Mga}{I}\theta = 0$ **(3/4pt)**

3- a) OG = a = $\frac{M\frac{\ell}{2} + Mx}{2M}$ = $\frac{\ell + 2x}{4}$ **(1/2pt)**

b) I = I(tige) + I(S) = $\frac{M\ell^2}{3} + Mx^2 = \frac{M(\ell^2 + 3x^2)}{3}$ **(1/2pt)**

4- $\omega^2 = \frac{2Mga}{I}$ and T = $\frac{2\pi}{\omega_0}$ = $2\pi\sqrt{\frac{I}{2Mga}} \Rightarrow T = \sqrt{\frac{8(1+3x^2)}{3(1+2x)}}$ **(1pt)**

B- 1) T'= $\frac{16,6}{10} = 1,66\,s$ **(1/4pt)**

2) a) T' = T **(1/2pt)**

b) i) T_1 = $T = \sqrt{\frac{8(1+3x^2)}{3(1+2x)}}$. for x = 0,3 m$\Rightarrow$ T_1 = 1,45 s **(1/2pt)**

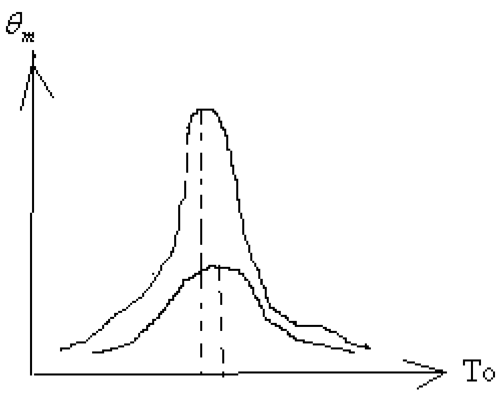
ii) T_2 is closer to T' que T_1, therefore $\theta_{m2} > \theta_{m1}$ **(1/2 pt)**

c) i) Resonance **(1/4pt)**

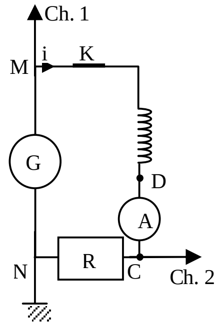
ii) At resonance T = T'$\Rightarrow$ $\sqrt{\frac{8(1+3x^2)}{3(1+2x)}} = 1,66$

$\Rightarrow$ $24x^2 - 16,53x - 0,27 = 0$ $\Rightarrow$ x = 0,7 m **(1pt)**

d) ; e) **(1/2pt)**



Second exercise : (7 pts)



1) Connection of the oscilloscope. **(1/2 pt)**

2) **a)** $u_{MN} = u(B) + u(R)$ **(1/4pt)**

$E = ri + L\frac{di}{dt} + Ri = (R+r)i + L\frac{di}{dt}$

b) i) In steady state, $i = cte = I_0 \Rightarrow \frac{di}{dt} = 0$

$\Rightarrow E = (R+r)\,I_0$ et $u_{MD} = rI_0$ **(1/2pt)**

ii) $R + r = \frac{E}{I_0} \Rightarrow R + r = 60\ \Omega$.

$U_0 = R\,I_0$, $U_0 = 5\times2 = 10\ V \Rightarrow R = 50\ \Omega$; $\Rightarrow r = 10\Omega$ **(1 pt)**

3) **a)** **i)** $u_{CN} = u_R = Ri \Rightarrow i = \frac{u_{CN}}{R}$ et $\frac{di}{dt} = \frac{1}{R} \times \frac{du_{CN}}{dt}$.

$E = (R+r) \times \frac{u_{CN}}{R} + \frac{L}{R}\frac{du_{CN}}{dt} \Rightarrow \frac{RE}{L} = \frac{du_{CN}}{dt} + \frac{(R+r)}{L}\ u_{CN}$ **(3/4pt)**

ii) at t = o, $u_{CN} = 0 \Rightarrow \left(\frac{du_{CN}}{dt}\right)_{t=0} = \frac{RE}{L}$ **(1/2 pt)**

iii) $\left(\frac{du_{CN}}{dt}\right)_{t=0}$ is the slope of the tangent to the curve of u_{CN} :

$\left(\frac{du_{CN}}{dt}\right)_{t=0} = \frac{U_0}{\tau} \Rightarrow \frac{RE}{L} = \frac{U_0}{\tau} \Rightarrow \tau = L\frac{U_0}{RE}$.

$\tau = \frac{LI_0R}{[R(R+r)I_0]} = \frac{L}{(R+r)}$. **(1 pt)**

b) Explanation of the used method. **(1/4pt)**

c) $L = (R+r)\,\tau = 60\times10^{-3}\ H = 60\ mH$. **(1/2 pt)**

4) $E_{max} = \frac{1}{2}L\,I_0^2 = \frac{1}{2}\ 60\times10^{-3}\times0{,}04 = 1{,}2\times10^{-3}\ J$. **(1/2 pt)**

5) $i = I_0\,(1 - e^{-t/\tau})$; $E = \frac{1}{2}L\,i^2 = \frac{1}{2}L\,I_0^2\,(1- e^{-t/\tau})^2$

$\frac{E}{E_{max}} = \frac{0{,}5LI_0^2(1-e^{-t/\tau})^2}{0{,}5LI_0^2} \geq 0{,}903 \Rightarrow (1- e^{-t/\tau}) \geq \sqrt{0{,}903}$

$\Rightarrow e^{-t/\tau} \leq 1-\sqrt{0{,}903} = 0{,}05 \Rightarrow \frac{-t}{\tau} \leq Ln\,(0{,}05)$

$\Rightarrow t \geq 3$ ms the minimum duration of closure is 3 ms. **(1 1/4pt)**

<u>Third exercise</u> : (7 pts)

A - 1) At the end of charging i = 0 $\Rightarrow u_C = E-Ri = E=U_0 = 10$ V **(1/2 pt)**

2) $W = ½ C(E)^2 = 5\times 10^{-5}$ J **(1/2pt)**

B- I) 1) Figure **(1/4pt)**

2) $u_C = u_{AM} = L\frac{di}{dt}$; $i = -\frac{dq}{dt} = -C\frac{du_C}{dt} = -C\,u'_C$ **;** $\frac{di}{dt} = -C\,u''_C \Rightarrow$

$LC\,u''_C + u_C = 0$ **(3/4pt)**

3) $LC\,u''_C + u_C = 0 \Rightarrow u''_C + \frac{1}{LC}u_C = 0 \Rightarrow$ The proper pulsation ω_0 of the oscillation is $(\omega_0)^2 = \frac{1}{LC} \Rightarrow$ the proper period is $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{LC}$ **(3/4 pt)**

4) (1/4 pt)

5) The oscillation is free un damped **(1/4 pt)**

II- 1) The oscillation is free damped **(1/4 pt)**

2) a) T = 2 ms **(1/4pt)**

b) $\frac{8,2}{10} = \frac{6,7}{8,2} = \frac{5,5}{6,7} = 0,82$ **= cte =** a **(1/2 pt)**

3) a) Si u_C is max. $\Rightarrow i = -C\frac{du_C}{dt} = 0 \Rightarrow$ the magnetic energy in the coil $E_{magn} = ½ Li^2$ is zero. The energy stored is in the electric form in the capacitor. $[E_{él} = ½ C(u_C)^2]$. **(3/4pt)**

b) $\frac{E_{(n+1)}}{E_n} = \frac{0,5C(u_{Cmax(n+1)})^2}{0,5C(u_{Cmax(n)})^2} = a^2$ **(3/4 pt)**

c) $\frac{E_{(n+1)}}{E_n} = e^{-\frac{r}{L}T} = a^2 \Rightarrow -\frac{rT}{L} = 2\ln a \Rightarrow \frac{r}{L} = \frac{-2\ln a}{T} = -2\frac{\ln 0,82}{0,002} = 198,45$

$\frac{4\pi^2}{T^2} = \frac{4\pi^2}{T_0^2} - \frac{1}{4}(\frac{r}{L})^2$ with $T_0^2 = 4\pi^2 LC = 4\pi^2 \times 10^{-6} L$, we get :

L = 0,1 H and r = 20 Ω. **(1 1/4pt)**

<u>Fourth exercise</u> : (6 ½ pts)

A- **1)** a) $({}^{230}_{Z}Th) \longrightarrow \left({}^{A}_{88}Ra\right) + {}^{4}_{2}He$

230 = A + 4 ⇒ A = 226; Z = 88 + 2 = 90. **(3/4 pt)**

b) $\Delta m = m({}^{230}_{Z}Th) - m\left({}^{A}_{88}Ra\right) - m({}^{4}_{2}He) = 5{,}1.10^{-3}$ u = 4,75 MeV/c^2

$E = \Delta m.c^2 = 4{,}75$ MeV. **(1 pt)**

2) The liberated energy appears in the form of kinetic energy of the particle α and electromagnetic energy of the radiation γ : $E = E_C + E(\gamma)$.

In the case where $E(\gamma) = 0$, we obtain $E = E_C = E_{C1} = 4{,}75$ MeV. **(1/2 pt)**

3) a) $E(\gamma) = \frac{hc}{\lambda} = 3{,}315\times10^{-14}$ J = 0,21 MeV **(1/2 pt)**

b) $E_{C2}({}^{4}_{2}He) = 4{,}75 - 0{,}21 = 4{,}54$ MeV. **(1/2 pt)**

4) a) Let x be the number of decays/s for each type. We get : $xE_{c1} + xE_{c2} = W$; but

$A_0 = 2x = 7{,}2\times10^8$ Bq. $\Rightarrow \frac{A_0}{2}(E_{c1} + E_{c2}) = W \Rightarrow W = 5{,}35\times10^{-4}$ j. **(1 1/4 pt)**

b) $n_0 = \frac{m}{M}N = \frac{6{,}02\times10^{23}}{230} = 2{,}62\times10^{21}$ nuclei ; $\lambda = \frac{A_0}{n_0} = 2{,}75\times10^{-13}\ s^{-1} = 86724\times10^{-10}\ year^{-1}$. **(1 pt)**

B- $A = A_0 e^{-\lambda t}$ ⇒ ln 0,12 = $-\lambda t$ ⇒ t = 244484 years. **(1 pt)**