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الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	

This exam is formed of four exercises in four pages numbered from 1 to 4.
The use of non-programmable calculator is allowed.

First exercise (7.5 points)

Response of an electric component submitted to a DC voltage

In order to study the response of the current in an electric component when submitted to a DC voltage, we use a coil of inductance L = 40 mH and of resistance r = 18 Ω, a capacitor of capacitance C = 100 μF, a resistor of resistance R = 2 Ω, a switch K and a DC generator delivering across its terminals a constant voltage E = 8 V.

A – Response of the electric component (R, L)

We connect the coil in series with the resistor across the terminals of the generator (Fig. 1).

At the instant $t_0 = 0$, we close K. The circuit thus carries a current i. With an oscilloscope, we display the variation of the voltage u_{AM} across the terminals of the resistor as a function of time (Fig. 2).

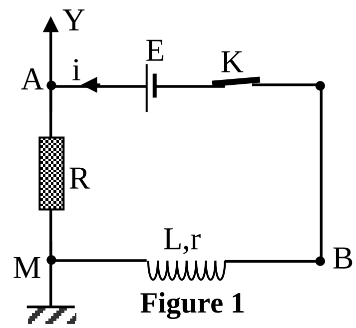


Figure 1

1) Express the voltage u_{AM} across the resistor and the voltage u_{MB} across the coil in terms of R, L, r, i and $\frac{di}{dt}$.
2) Derive the differential equation in i.
3) The solution of this differential equation is of the form:

   $i = I_0(1- e^{-\frac{t}{\tau}})$.

   a) Show that $I_0 = \frac{E}{R+r}$ and $\tau = \frac{L}{R+r}$.
   b) Calculate the values of I_0 and τ.
4) Using figure 2, determine the values of I_0 and that of τ.

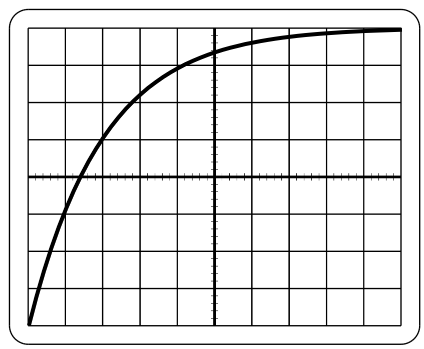


Figure 2
Horizontal sensitivity : 1 ms/div
Vertical sensitivity: 0.1 V/div

B – Response of the electric component (R,C)

We replace, in the previous circuit, the coil by the capacitor (Fig. 3).

At $t_0 = 0$, we close K. The circuit thus carries a current i. With the oscilloscope, we display the variation of the voltage u_{AM} as a function of time (Fig. 4).

1) Express the current i in terms of C and $\frac{du_C}{dt}$, where u_C is the voltage u_{MB} across the terminals of the capacitor.
2) Using the law of addition of voltages, show that the differential equation in i is of the form: $RC\frac{di}{dt} + i = 0$.

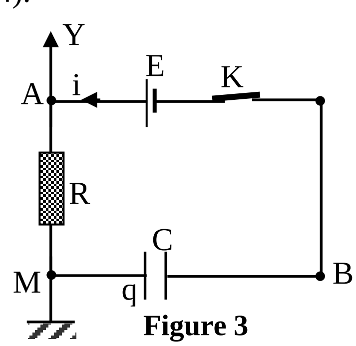


Figure 3

3) The solution of this differential equation is of the form:

$i = I_1 e^{-\frac{t}{\tau_1}}$. Determine, in terms of E, R and C, the expressions of the two constants I_1 and τ_1 and calculate their values.

4) Referring to figure 4, determine the value of I_1 and that of τ_1.

C – In each of the two previous circuits, we replace the resistor by a lamp. Explain the variation of the brightness of the lamp in each circuit.

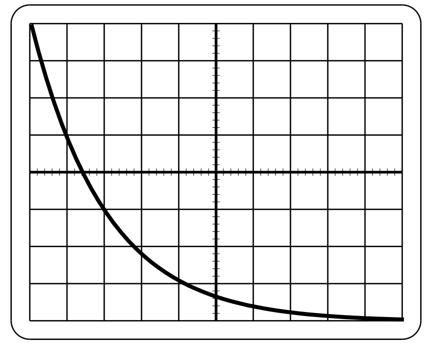


Figure 4

Horizontal sensitivity : 0.1 ms/div
Vertical sensitivity: 1 V/div

Second exercise (7.5 points)

(R,L,C) series circuit

Consider a capacitor of capacitance C = 5 μF, a resistor of resistance R = 40 Ω and a coil of inductance L and of resistance r, connected in series across the secondary of an ideal transformer.

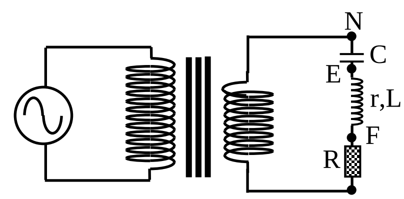


Figure 1

A – Physical quantities of the transformer

The primary coil of the transformer is connected to the mains (220 V; 50 Hz) (Fig.1). The secondary of the transformer delivers across its terminals a voltage: $u_{NM} = 3\cos\omega t$ (u in V ; t in s).

The circuit thus carries an alternating sinusoidal current $i = I_m\cos(\omega t - \varphi)$.

The secondary coil has 15 turns and cannot withstand a current of effective value greater than 10 A.

1) Give the value of the frequency of the alternating sinusoidal voltage across the secondary coil.
2) Determine the number of turns of the primary coil. Take $\sqrt{2} = 1.4$.
3) Calculate the maximum effective value of the current that the primary coil can withstand.

B – Determination of L and r

An oscilloscope, connected in the previous circuit, allows us to display on the channel Y_1 the voltage $u_1 = u_{NM}$ and on the channel Y_2 the voltage $u_2 = u_{FM}$ across the terminals of the resistor.

1) Redraw the circuit of figure 1 and show the connections of the oscilloscope.
2) The sensitivities of the oscilloscope are:
   Horizontal sensitivity: 4 ms/div
   Vertical sensitivity on both channels Y_1 and Y_2: 1 V/div.
   Using the waveforms of figure 2,
   show that $i = 0.05\cos(100\pi t - 0.2\pi)$; (i in A, t in s).
3) Calculate the average power consumed by the component NM.
4) Deduce the value of the resistance r of the coil.
5) Knowing that $u_{NM} = u_{NE} + u_{EF} + u_{FM}$ is verified for any value of time t, determine the value of L.

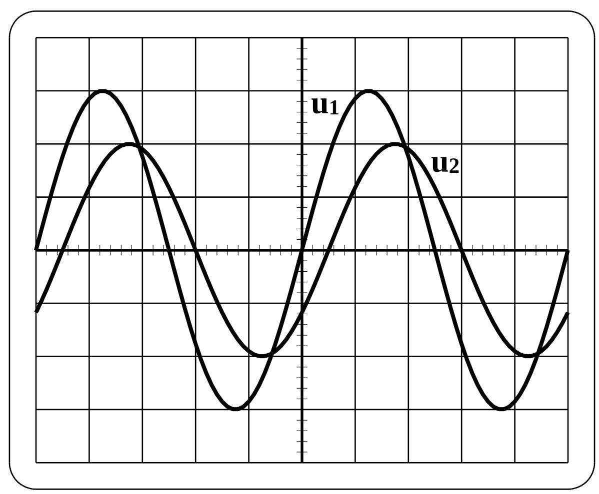
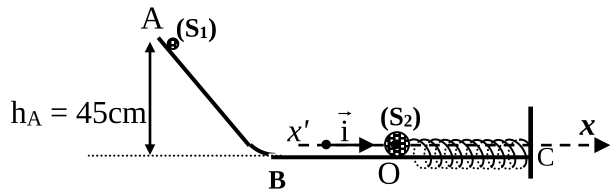


Figure 2

<u>Third exercise</u> (7.5 points)

Determination of the stiffness constant of a spring

To determine the stiffness constant k of a spring we attach to its extremity a solid (S_2), of mass m_2 = 200 g, which can slide without friction on the horizontal part BC of a track ABC situated in a vertical plane, the other extremity of the spring is fixed at C.



Another solid (S_1), of mass m_1 = 50 g, is released without initial velocity from a point A of the curved part of the track.

Point A is situated at a height h_A = 45 cm from the horizontal part of the track.

(S_2), initially at rest at point O, is thus hit by (S_1). (S_1) and (S_2) are supposed to be point masses.

The horizontal plane passing through BC is taken as a gravitational potential energy reference.

Take: g = 10 ms^{-2} , $0.32\pi = 1$. Neglect all frictional forces.

1) Determine the value V_1 of the velocity $\vec{V_1}$ of (S_1) just before colliding (S_2).

2) After collision, (S_1) remains in contact with (S_2) and the two solids form a solid (S) of center of inertia G and of mass $M = m_1 + m_2$. Thus G performs oscillations around O with amplitude 3 cm on the axis x'Ox of origin O and unit vector $\vec{i}$.

a) Show that the value of the velocity $\vec{V_0}$ of G just after the collision is equal to 0.6 m/s.

b) Let x and v be respectively the abscissa and the algebraic value of the velocity of G at an instant t after

the collision. The instant of collision at O is considered as an origin of time $t_0 = 0$.

i) Write down, at an instant t, the expression of the mechanical energy of the system (S, spring, Earth) in terms of k, x , M and v.

ii) Deduce the second order differential equation in x that describes the motion of G.

iii) The time equation of oscillation of (S) is given by: $x = X_m\sin(\omega_0 t + \varphi)$. Determine the value of φ and the expressions of the constants X_m and ω_0 in terms of k, M and V_0.

iv) Deduce the value of the stiffness constant k of the spring.

3) In reality friction is not neglected. To ensure the value of k, the extremity C of the spring is attached to a vibrator of adjustable frequency f and which can vibrate in the same direction of the spring. We notice that the amplitude of the oscillations of (S) varies with f and attains a maximum value for f = 3.2 Hz.

a) Name the physical phenomenon that takes place when f = 3.2 Hz.

b) Calculate the value of k.

<u>Fourth exercise</u> (7.5 points)

The radionuclide Potassium 40

The isotope of potassium $^{40}_{19}K$, is radioactive and is β^+ emitter; it decays to give the daughter nucleus argon $^{A}_{Z}Ar$. The object of this exercise is to study the decay of potassium 40.

Given:

masses of nuclei: m ($^{40}_{19}K$) = 39.95355 u ; m($^{A}_{Z}Ar$) = 39.95250 u ;

masses of particles: m ($^{0}_{1}e$) = 5.5×10^{-4} u; m (neutrino) ≈ 0 ;

Avogadro's number: N = 6.02×10^{23} mol^{-1}; 1 u = 931.5 MeV/c^2 ;

Radioactive period of $^{40}_{19}K$: T = 1.5×10^{9} years; molar mass of $^{40}_{19}K$ = 40 g mol^{-1}.

1 MeV = 1.6×10^{-13} J.

A – Energetic study of the decay of potassium 40

1) Energy liberated by one decay

a) Write down the equation of the decay of one potassium 40 nucleus and determine Z and A.
b) Calculate, in MeV, the energy E_1 liberated by this decay.
c) The daughter nucleus is supposed to be at rest. The energy carried by β^+ is, in general, smaller than E_1. Why?

2) Energy received by a person

The mass, of potassium 40 at an instant t, in the body of an adult is, on the average, equal to 2.6×10^{-3} % of its mass.

An adult person has a mass M = 80 kg.

a) i) Calculate the mass m of potassium 40 contained in the body of that person at the instant t.
ii) Deduce the number of potassium 40 nuclei in the mass m at the instant t.

b) i) Calculate the radioactive constant λ of potassium 40.
ii) Deduce the value of the activity A of the mass m at the instant t.

c) Deduce, in J, the energy E liberated by the mass m per second.

B – Dating by potassium 40

Certain volcanic rocks contain potassium and part of it, is potassium 40. At the instant of its formation (t_0 = 0), the number of nuclei of potassium 40 is N_0 in the volcanic rock and that of argon is zero. At the instant t, the rock contains respectively N_K and N_{Ar} nuclei of potassium 40 and of argon 40.

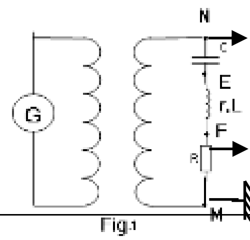
1) a) Write down the expression of N_K **,** that explains the law of radioactive decay, as a function of time.
b) Deduce the expression of N_{Ar} as a function of time.

2) A geologist analyzes a volcanic rock. He notices that the number of argon 40 nuclei is twice less than the number of potassium 40 nuclei in this rock. Determine the age of this rock.

First exercise (7.5 points)

Part of the Q	Answer	Mark
A.1	$u_{AM} = Ri$ and $u_{MB} = L\dfrac{di}{dt} + ri$.	0.5
A.2	We have $E = Ri + L\dfrac{di}{dt} + ri \Rightarrow i + \dfrac{L}{R+r}\dfrac{di}{dt} = \dfrac{E}{R+r}$.	0.75
A.3.a	$\dfrac{di}{dt} = \dfrac{I_0}{\tau}e^{-\frac{t}{\tau}}$; $I_0 - I_0 e^{-\frac{t}{\tau}} + \dfrac{L}{R+r}\dfrac{I_0}{\tau}e^{-\frac{t}{\tau}} = \dfrac{E}{R+r}$ $\Rightarrow I_0 = \dfrac{E}{R+r}$ and $\dfrac{L}{R+r}\dfrac{I_0}{\tau} - I_0 = 0$; let $\tau = \dfrac{L}{R+r}$.	1.25
A.3.b	$I_0 = \dfrac{8}{18+2} = 0.4$ A and $\tau = \dfrac{0.04}{18+2} = 2\times10^{-3}$ s = 2 ms.	0.5
A.4	From graph 2: $u_R(max) = 0.1\times8 = 0.8$ V and $u_R(max) = R\times I_0$ $\Rightarrow I_0 = \dfrac{u_R(max)}{R} = 0.4$ A. Also, for $t = \tau$, $u_R = 0.63\, u_R(max) = 0.5$ V which corresponds to $\tau = 2$ divisions, $\tau = 2$ ms.	1.00
B.1	$i = \dfrac{dq}{dt} = C\dfrac{du_C}{dt}$.	0.25
B.2	$E = u_{AM} + u_{MB} \Rightarrow E = u_C + Ri$. By deriving with respect to time: $0 = \dfrac{du_C}{dt} + R\dfrac{di}{dt} \Rightarrow \dfrac{i}{C} + R\dfrac{di}{dt} = 0$ Thus : $RC\dfrac{di}{dt} + i = 0$	0.75
B.3	$i = I_1 e^{-\frac{t}{\tau_1}}$. For $t_0 = 0$, $u_C = 0$ and $i = I_1 \Rightarrow E = 0 + RI_1$ $\Rightarrow I_1 = \dfrac{E}{R} = \dfrac{8}{2} = 4$ A. $\dfrac{di}{dt} = -\dfrac{I_1}{\tau_1}e^{-\frac{t}{\tau_1}}$; by replacing: $-RC\dfrac{I_1}{\tau_1}e^{-\frac{t}{\tau_1}} + I_1 e^{-\frac{t}{\tau_1}} = 0$ $\Rightarrow -RC\dfrac{I_1}{\tau_1} + I_1 = 0 \Rightarrow \tau_1 = RC = 2\times10^0\times10^{-6} = 2\times10^{-4} = 0.2$ ms.	1
B.4	$u_R(max) = 8\text{ V} = RI_1 \Rightarrow I_1 = 8/2 = 4$ A and for $t = \tau_1$, $u_R = 0.37\, u_R(max) = 3\text{ V} \Rightarrow \tau_1 = 0.2$ ms.	0.5
C	In A: after closing the switch the brightness of the lamp increases and reaches after a very short time a stable brightness. In B : at the instant of closing the switch the lamp shines then the brightness decreases and vanishes after a short time	1

Second exercise (7.5 points)

Part of the Q	Answer	Mark
A.1	f = 50 Hz	0.5
A.2	$\frac{U_2}{U_1} = \frac{N_2}{N_1} \Rightarrow \frac{3/\sqrt{2}}{220} = \frac{15}{N_1} \Rightarrow N_1 = 1540$ turns.	0.75
A.3	$\frac{I_2}{I_1} = \frac{N_1}{N_2} \Rightarrow \frac{10}{I_1} = \frac{1540}{15} \Rightarrow I_1 = 97$ mA	0.75
B.1		0.25
B.2	T = 5 div$\times$4 ms/div = 20 ms = 0.02 s $\Rightarrow \omega = \frac{2\pi}{0,02} = 100\pi$ rad/s. $(U_R)_{max} = RI_{max} \Rightarrow I_{max} = \frac{2}{40} = 0.05$ A. $\varphi = 0.5\times 2\pi/5 = 0.2\,\pi$ rad. i is in lag on $u_{NM} \Rightarrow$ i = 0.05 cos(100πt – 0.2 π)	1.5
B.3	$P = UI\cos\varphi = \frac{3}{\sqrt{2}} \times \frac{0.05}{\sqrt{2}} \times \cos 0.2\pi = 0.061$ W.	0.75
B.4	$P = R_{total} I^2 \Rightarrow R_{totale} = \frac{0.061}{(0.05/\sqrt{2})^2} = 48.8\ \Omega = R + r = 40 + r$ => r = 8.8 Ω	1
B.5	$u_{NE} = u_C$ = 1/C primitive (i) = 100/ π sin(1ooπt – 0.2 π) u_{EF} = ri + Ldi/dt u_{EF} = 8,8$\times$0.,05cos(100πt – 0.2 π) - L$\times$5 π sin(100πt – 0,.2 π). u_{FM} = Ri = 2 cos(100πt – 0.2 π). 3cosωt = 100/ π sin(ωt – 0.2 π) + 8,8$\times$0.05cos(100πt – 0.2 π) - L$\times$5 π sin(100πt– 0.2 π) + 2 cos(100πt – 0.2 π). For t = 0, we obtain L = 2.15 H.	2

Third exercise (7.5 points)

Part of the Q	Answer	Mark
1	Conservation of mechanical energy between A and B: $m_1gh_A+0 = 0+ \frac{1}{2} m_1 V_1^2$; $V_1 = \sqrt{2gh_A} = \sqrt{2\times10\times0.45} = 3$ m/s .	1.25
2.a	Conservation of linear momentum: $m_1 \vec{V}_1 + \vec{0} = (m_1 + m_2) \vec{V}_0$; projection : $V_0 = \frac{m_1}{m_1+m_2} V_1 = \frac{0.05}{0.05+0.2} 3 = 0.6$ m/s	1.00
2.b. i	$ME = \frac{1}{2} M v_G^2 + \frac{1}{2} kx^2$; $(M = m_1 + m_2)$.	0.50
2.b.ii	ME is conserved: Derivative w.r.t time $\frac{d(ME)}{dt} = 0$ $\Rightarrow Mv\dot{v} + kx\dot{x} = 0 \Rightarrow \ddot{x} + \frac{k}{M}x = 0$	1.00
2.b.iii	$\dot{x}$ $x' = \omega_0 X_m\cos(\omega_0 t + \varphi)$ and $\ddot{x} = -\omega_0^2 .X_m\sin(\omega_0 t + \varphi)$. By replacing : $-\omega_0^2 X_m\sin(\omega_0 t + \varphi) + \frac{k}{M} X_m\sin(\omega_0 t + \varphi) \Rightarrow \omega_0^2 = \frac{k}{M} \Rightarrow \omega_0 = \sqrt{\frac{k}{M}}$; At t = 0: $x = 0 \Rightarrow X_m\sin\varphi = 0 \Rightarrow \varphi = 0$ or π. At t = 0: $v = V_0 \Rightarrow \omega_0 X_m\cos\varphi = V_0 > 0 \Rightarrow \varphi = 0$, $X_m = \frac{V_0}{\omega_0} = V_0\sqrt{\frac{M}{k}}$	2.00
2.b.iv	$X_m = X_0 = V_0\sqrt{\frac{M}{k}} \Rightarrow k = \frac{V_0^2 M}{X_m^2} = \frac{0.36\times0.25}{0.03^2} = 100$ N/m.	0.75
3.a	Resonance.	0.25
3.b	$\omega_0 = \omega = 2\pi f = \sqrt{\frac{k}{M}}$; $4\pi^2 f^2 = \frac{k}{M} \Rightarrow k = 4\pi^2 f^2 M = 100$ N/m	0.75

Fourth exercise (7.5 points)

Part of the Q	Answer	Mark
A.1.a	${}^{40}_{19}K \rightarrow {}^{A}_{Z}Ar + {}^{0}_{1}e + {}^{0}_{0}\nu$. Z = 18; A = 40.	z0.75
A.1.b	$\Delta m = 39.95355 - 39.95250 - 5.5\times 10^{-4} = 5\times10^{-4}$ u. $E_1 = mc^2 = 5\times10^{-4}\times 931.5\ MeV/c^2 \times c^2 = 0.47$ MeV.	1.00
A.1.c	Because $E_1 = E(\beta^+) + E({}^{0}_{0}\nu) + E(\gamma)$	0.50
A.2.a.i	$m = \dfrac{80\times2.6\times10^{-3}}{100} = 2.1\times10^{-3}\ kg = 2,1\ g$	0.50
A.2.a.ii	$N = \dfrac{m}{M}$ **N =** 3.16×10^{22} nuclei.	0.50
A.2.b.i	$\lambda = \dfrac{0.693}{1.5\times10^9\times365\times24\times3600} = 1.46\times10^{-17}\ s^{-1}$	0.5
A.2.b.ii	$A = \lambda N = 1.46\times10^{-17}\times3.16\times10^{22} = 4.61\times10^5$ Bq	0.75
A.2.c	The energy received in each second: $E = 4.16\times10^5\times0.47 = 2.17\times10^5$ MeV $= 3.47\times10^{-8}$ J.	07.5
B.1.a	$N_K = N_0 e^{-\lambda t}$	0.50
B.1.b	$N_{Ar} = N_0 - N_K = N_0(1-e^{-\lambda t})$	0.50
B.2	$\dfrac{N_{Ar}}{N_K} = \dfrac{1}{2} \Rightarrow \dfrac{1 - e^{-\lambda t}}{e^{-\lambda t}} = \dfrac{1}{2} \Rightarrow e^{\lambda t} = \dfrac{3}{2}$ $\Rightarrow t = \dfrac{T}{0.693}\ln\dfrac{3}{2} \Rightarrow t = 8.8\times10^8$ years	1.25