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الاسم: الرقم:	مسابقة في مادة الفيزياء المدة : ثلاث ساعات	

<u>This exam is formed of four exercises in four pages</u>

<u>The use of non-programmable calculator is recommended</u>

<u>First Exercise</u> (7.5 points) Interference of light

Consider Young's double slit apparatus that is represented in the adjacent figure 1. S_1 and S_2 are separated by a distance a = 1mm.

The planes (P) and (E) are at a distance D = 1m. I is the midpoint of S_1S_2 and O the orthogonal projection of I on (P). On the perpendicular to IO at point O and parallel to S_1S_2, a point M is defined by its abscissa OM = x.

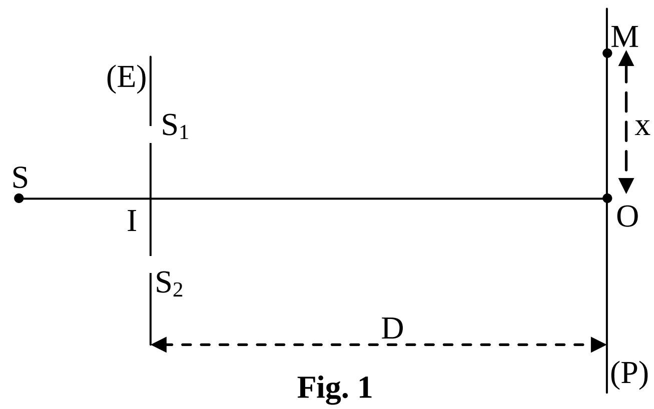


Fig. 1

1) S_1 and S_2, illuminated by two lamps, emit synchronous radiations. Do we observe interference fringes on the screen? Why?

2) S_1 and S_2 are illuminated by a point source S put on IO. S emits a monochromatic radiation of wavelength λ in vacuum (or in air).

- **a)** Is the fringe obtained at O bright or dark? Why?
- **b)** Give, at point M ,the expression of the optical path difference δ between the two radiations emitted by S, one passing through S_1 and the other through S_2, in terms of D, x and a.
- **c)** Derive the relation giving the abscissas of the centers of the bright fringes and that giving the abscissas of the centers of the dark fringes.
- **d)** For x = 2.24 mm, M is at the center of the fourth bright fringe (bright fringe of order 4). Calculate λ.

3) The source S emits now white light.

- **a)** At O, we observe a white light. Why ?
- **b)** Calculate the wavelengths of the visible radiations that give at M, of abscissa OM = x = 2.24mm, dark fringes.

  Visible spectrum: $0.400\ \mu m \leq \lambda \leq 0.800\ \mu m$.

<u>Second Exercise</u> (7.5 points) Determination of the half-life of Polonium 210

Polonium 210 nucleus ($^{210}_{84}Po$) is an α emitter, and it is the only polonium isotope that exists in nature; it was found by Pierre Curie in 1898 in an ore. It is also obtained from the decay of a bismuth 210 nucleus ($^{210}_{83}Bi$).

Masses of the nuclei: m (Bi) = 209.938445 u ; m(Po) = 209.936648 u

mass of the electron : m_e = 0.00055 u

1 u = 931.5 MeV / c^2 = 1.66 10^{-27} kg.

Here is a part of the periodic table of elements: $_{81}Th$; $_{82}Pb$; $_{83}Bi$; $_{84}Po$; $_{85}At$; $_{86}Rn$.

A –The polonium 210

1) **a)** Write down the equation of the decay of bismuth 210.
b) Identify the emitted particle and specify the type of this decay.

2) Calculate the energy liberated by this decay.

3) The decay of the bismuth 210 nucleus is accompanied with the emission of a γ photon of energy $E(\gamma) = 0.96$ MeV and an antineutrino of energy 0.02 MeV. Knowing that the daughter nucleus is practically at rest, calculate the kinetic energy of the emitted particle.

B – Half-life of polonium 210

1) **a)** Write down the equation of the decay of polonium 210.
   b) Identify the daughter nucleus.

2) In order to determine the radioactive period T (half-life) of ${}^{210}_{84}\text{Po}$, we consider a sample of this isotope containing N_o nuclei at the instant $t_o = 0$. Let N be the number of the non-decayed nuclei at an instant t.
   a) Write down the expression of the law of radioactive decay.
   b) Determine the expression of $-\ell n\left(\frac{N}{N_0}\right)$ as a function of t .

3) A counter allows to obtain the measurements that are tabulated in the following table :

t (days)	0	40	80	120	160	200	240
$\frac{N}{N_0}$	1	0.82	0.67	0.55	0.45	0.37	0.30
$-\ell n\left(\frac{N}{N_0}\right)$	0		0.4		0.8		1.2

   a) Complete the table.
   b) Trace, on the graph paper, the curve giving the variation of $-\ell n\left(\frac{N}{N_0}\right)$ as a function of time.

   Scale: 1 cm on the abscissa axis corresponds to 40 days.
   1 cm on the ordinate axis corresponds to 0.2.

   c) Is this curve in agreement with the expression found in the question (B – **2**, b) ? Justify.
   d) **i)** Calculate the slope of the traced curve.
   ii) What does this slope represent for the polonium 210 nucleus?
   iii) Deduce the value of T.

<u>Third Exercise</u> (7.5 points) Exchanged Energy

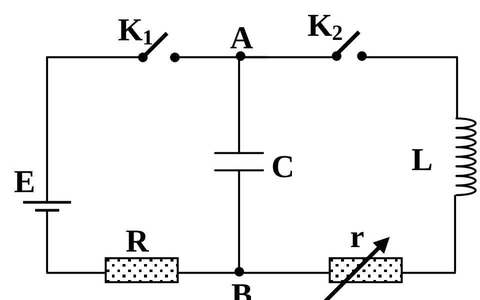


Fig. 1

We connect up the circuit formed of a resistor of resistance $R = 2.2$ kΩ, an ideal generator of e.m.f $E = 8$ V, a coil of inductance $L = 0.8$ H and of negligible resistance ,a resistor of adjustable resistance r and two switches K_1 and K_2 (Fig.1).

A – (RC) Series circuit

At an instant taken as an origin of time, ($t_0 = 0$), we close the switch K_1, and K_2 remains open.

We study the charging of the capacitor through the variation of the voltage $u_{AB} = u_C$ as a function of time.

1) Show that the differential equation in u_C is:

$$E = u_C + RC\frac{du_C}{dt}.$$

2) Knowing that $u_C = E\,(1- e^{\frac{-t}{\tau}})$ is the solution of this differential equation, determine the expression of the time constant τ in terms of R and C.

3) The curve of figure 2 shows the variation of u_C as a function of time. Using this curve, determine the time constant τ (indicating the method used).

4) Calculate the value of C.

5) **a)** Give, in ms, the duration t_1 at the end of which the voltage across the capacitor will no longer practically vary.

b) Calculate the charge of the capacitor and the energy W_0 stored at the end of the duration t_1.

B – (L,C) series circuit

We give r the value zero. The voltage across the capacitor is 8 V. At an instant taken as an origin of time ($t_0 = 0$), we open the switch K_1 and we close the switch K_2.

1) Derive the differential equation that describes the variation of the voltage u_C as a function of time.
2) The circuit is the seat of electric oscillations of proper period T_0. The solution of this differential equation is: $u_C = E\cos(\frac{2\pi}{T_0}t)$. Determine the value of T_o .
3) Trace the shape of the curve representing the variation of u_C as a function of time.
4) Specify the energy exchanges that take place in the circuit.

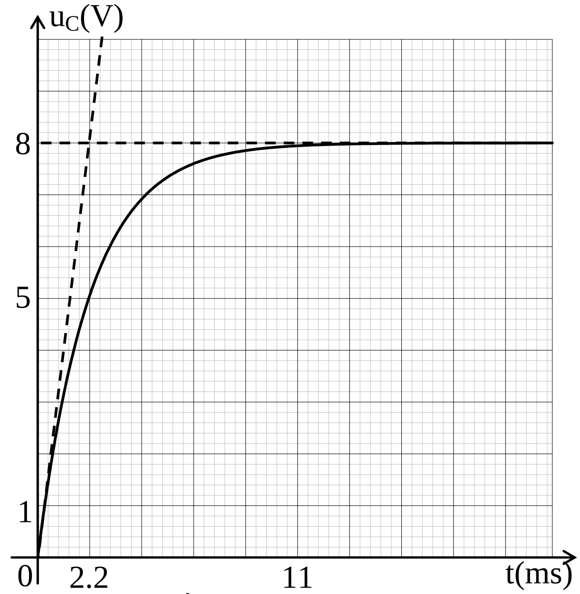


Fig.2

C – (r, L, C) series circuit

We give r a certain value. The voltage across the terminals of the capacitor is 8 V. We open K_1 and we close K_2 at the instant $t_0 = 0$. The waveform of figure 3 shows the variation of the voltage u_C as a function of time.

1) Specify the energy exchanges that take place in the circuit.
2) **a)** Referring to figure 3, find the pseudo-period T of the electric oscillations.
   b) Compare T and T_0.
3) At the end of the duration $t_n = nT$ (n being a whole number), the energy dissipated by Joule's effect is 98.6 % of the energy W_0 initially stored in the capacitor.
   a) At the instant $t_n = nT$, the energy stored in the circuit is purely electric. Why?
   b) We denote by W_0 and W_n the electric energy of the oscillator at the instants t_0 and t_n respectively. Calculate W_n.
   c) Determine n.

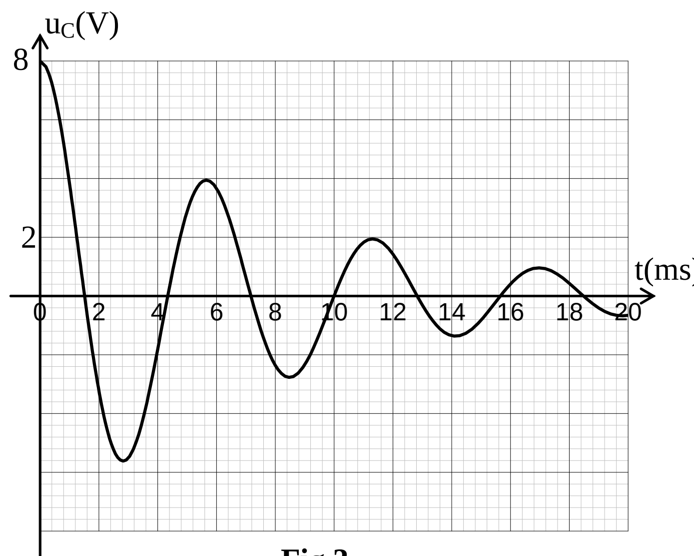


Fig.3

<u>Fourth Exercise</u> (7.5 points) Compound Pendulum

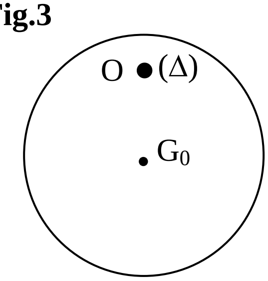


Fig 1

The aim of this exercise is to study the variation of the proper period T of a compound pendulum as a function of the distance a , of adjustable value, separating the axis of oscillation from the center of mass of this pendulum, and to show evidence of some properties associated to this distance a .

We consider a homogeneous disc (D) of mass m = 200g, free to rotate without friction around a horizontal axis (Δ) perpendicular to its plane through a point O (Fig. 1).

I_0 is the moment of inertia of (D) about the axis (Δ_0) parallel to (Δ) and passing through its center of mass G and I its moment of inertia about the axis (Δ), (Δ_0) being at a distance a = OG from (Δ) , so that: $I = I_0 + ma^2$.

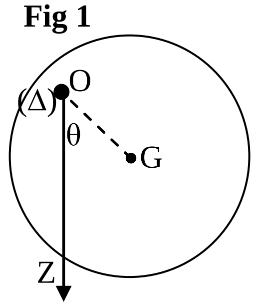


Fig 2

The gravitational potential energy reference is the horizontal plane passing through the center of mass G_0 of (D) when (D) is in the position of stable equilibrium (Fig.1).
(D) is made to oscillate around (Δ) and we measure the value of the proper period corresponding to each value of a.

Take: $g = 10\ m/s^2$; $\pi^2 = 10$;

For small angles (θ in radian); $1 - \cos\theta = \frac{\theta^2}{2}$ and $\sin\theta = \theta$.

A – Theoretical study

(D) is shifted from its stable equilibrium position by a small angle θ_m and is then released from rest at the instant $t_0 = 0$. (D) thus oscillates around the axis (Δ) with a proper period T.

At an instant t, the angular abscissa of the pendulum is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$ (Fig. 2).

1) Write down, at the instant t, the expression of the mechanical energy of the system (pendulum, Earth) in terms of I, m, a, g, θ and θ'.
2) **a)** Derive the second order differential equation in θ that describes the motion of (D).
   b) Deduce that the expression of the period T of this pendulum is given by: $T = 2\pi\sqrt{\frac{I}{mga}}$.
3) T_1 and T_2 are respectively the periods of the pendulum when it oscillates around (Δ) that passes successively through O_1 and O_2 where $O_1G = a_1$ and $O_2G = a_2$. The oscillations have the same period ($T_1 = T_2$). I_1 and I_2 are respectively the moments of inertia of the pendulum around (Δ) that passes successively through O_1 and O_2.
   a) i) Find a relation among I_1, I_2, a_1 and a_2.
   ii) Deduce that $I_0 = m\, a_1 a_2$.
   b) The proper period T' of a simple pendulum of length ℓ, for oscillations of small amplitude, is given by the expression $T' = 2\pi\sqrt{\frac{\ell}{g}}$.
   Show that, when the value of T' is equal to that of T_1, we obtain $\ell = a_1 + a_2$.

B – Experimental study

We measure the value of the period T of the pendulum for each value of a. The obtained measurements allow us to trace the curve giving the variation of T as a function of a. The straight line of equation T = 1.1s intersects this curve in two points A and B. (Fig. 3)

1) **a)** Referring to the curve, give the values of a_1 and a_2 corresponding to the period T = 1.1 s.
   b) Deduce the value of I_0 and that of ℓ.
2) According to the curve of figure 3, T takes a minimum value ($T_{min} = 1.05$ s) for a certain value a' of a.
   a) Give, using the curve, the value of a' corresponding to T_{min}.
   b) Find, by calculation, the value of a' and that of T_{min}.

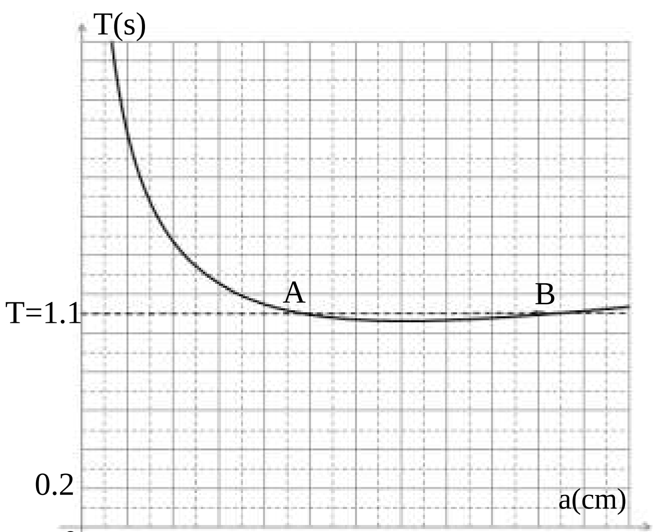


Fig.3

First exercise

1) We don't observe interference fringes since the two sources are not coherent. **(½)**

a) The two vibrations reach O in phase, the optical path difference at O is equal to zero :
$= (SS_1 + S_1O) - (SS_2 + S_2O) = 0$ (¾)

b) $\delta = \dfrac{ax}{D}$ **(½)**

c) Bright fringes : $\delta = k\lambda \Rightarrow \dfrac{ax}{D} = k\lambda \Rightarrow x = k\dfrac{\lambda D}{a}$

Dark fringes: $\delta = (2k+1)\dfrac{\lambda}{2} \Rightarrow \dfrac{ax}{D} = (2k+1)\dfrac{\lambda}{2}$

$\Rightarrow x = (2k+1)\dfrac{\lambda D}{2a}$ **(1½)**

d) $OM = x = 4i = 4\dfrac{\lambda D}{a}$

$\Rightarrow \lambda = \dfrac{ax}{4D} = \dfrac{1\times 2.24}{4\times 10^3} = 0.56\times 10^{-3}$ mm = 560 nm **(1¼)**

2) **a)** Every radiation gives at O bright fringe and because of superposition of all colors $\Rightarrow$ white light. **(1)**

b) $\dfrac{ax}{D} = (2k+1)\dfrac{\lambda}{2} \Rightarrow \lambda = \dfrac{2ax}{(2k+1)D} = \dfrac{2\times 1\times 2,24}{(2k+1)\times 10^3} = \dfrac{4.48\times 10^{-3}}{(2k+1)}$ mm $= \dfrac{4.48}{(2k+1)}$ µm

$0.4\mu m \le \lambda_V \le 0.8\mu m \Rightarrow 0.4 \le \dfrac{4,48}{(2k+1)} \le 0.8 \Rightarrow 5.6 \le (2k+1) \le 11.2 \Rightarrow (2k+1) \in \{7, 9, 11\}$

(2k + 1)	7	9	11
λµm	0.64	0.497	0.407

(2)

Second exercise

A – 1) a) $^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + {}^{a}_{z}X$

Coservation laws : $210 = a + 0 \Rightarrow a = 0$

$83 = 84 + z \Rightarrow z = -1$ **(½)**

b) The emitted particle is electron $^{0}_{-1}e$; $^{210}_{83}Bi$ is a β^{-1} emitter **(¼)**

2) $E_{libe} = m \times c^2$ where m is the mass defect

$\Delta m = m_{before} - m_{after} = m(Bi) - m(Po) - m(e)$
$\Delta m = 209.938445\ u - 209.936648\ u - 0.00055\ u = 1.247\times 10^{-3}\ u$

$E_{lib} = 1.247\times 10^{-3}\times 931.5\ \dfrac{MeV}{c^2}\times c^2 = 1.16$ MeV. **(1)**

3) $E_{lib} = E(Po) + E(\ ) + E(e) + E(\ )$

$\Rightarrow E_c = E(e) = 1.16 - 0.96 - 0.02 = 0.18$ MeV **(¾)**

B – 1) a) $^{210}_{84}Po \quad {}^{A}_{Z}X + {}^{4}_{2}He$

Conservation laws : $210 = A + 4 \Rightarrow A = 206$

$84 = Z + 2 \Rightarrow Z = 82$ **(½)**

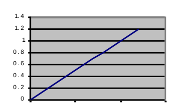
b) $^{a}_{z}X$ is $^{206}_{82}Pb$ **(¼)**

2) a) $N = N_o e^{-\lambda t}$ where λ is the radioactive constant **(½)**

b) $\dfrac{N}{N_0} = e^{-\lambda t}$ $\quad \ell n\left(\dfrac{N}{N_0}\right) = -\lambda t \Rightarrow -\ell n\left(\dfrac{N}{N_0}\right) = \lambda t$ **(½)**

3) a) missing values in the table : 0.20 ; 0.60 ; 1 **(½)**

b) **(1¼)**



c) The curve is a straight line passing through the origin is an agreement with $-\ell n\left(\dfrac{N}{N_0}\right) = \lambda t$. **(¼)**

d) i) $\lambda = \dfrac{0.6}{120} = 5 \quad 10^{-3}\ day^{-1}$ **(½)**

ii) The slope of this line is the radioactive constant λ of polonium 210. **(¼)**

iii) $T = \dfrac{\ell n 2}{\lambda} = 138.6$ days. **(½)**

Third exrcise

A – 1) $E = u_C + Ri$, $i = C\dfrac{du_C}{dt} \Rightarrow E = u_C + RC\dfrac{du_C}{dt}$ **(½)**

2) $u_C = E(1 - e^{-\frac{t}{\tau}}) \Rightarrow \dfrac{du_C}{dt} = \dfrac{E}{\tau}e^{-\frac{t}{\tau}} \Rightarrow E = E(1 - e^{-\frac{t}{\tau}}) + RC\dfrac{E}{\tau}e^{-\frac{t}{\tau}} \Rightarrow$

$\dfrac{RC}{\tau} = 1 \Rightarrow \tau = RC.$ **(¾)**

3) At instant $t = \tau$, $u_c = 0.63E = 0.63 \times 8 = 5.04$ V,

From graph : $t = \tau = 2.2$ ms for $u_c = 5.04$ V. **(½)**

4) $\tau = RC = 2.2\times10^{3} = 2.2\times10^{-3} \Rightarrow C = 10^{-6}F = 1\ \mu F.$ **(½)**

5) a) During $t_1 = 5\tau = 11$ ms. **(¼)**

b) $Q = CE = 8\times10^{-6}$ C ; $W_0 = ½ CE^2 = 32\times10^{-6}$ J. **(1)**

B – 1) $u_C = L\dfrac{di}{dt}$, $i = -\dfrac{dq}{dt} = -C\dfrac{du_C}{dt}$, $\Rightarrow u_C = -LC\ddot{u}_C \Rightarrow u_C + LC\ddot{u}_C = 0$ **(½)**

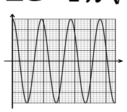
2) $u_C = E\cos(\dfrac{2\pi}{T_0}t) \Rightarrow \dot{u}_C = -E(\dfrac{2\pi}{T_0})\sin(\dfrac{2\pi}{T_0}t)$

$\Rightarrow \ddot{u}_C = -E(\dfrac{2\pi}{T_0})^2\cos(\dfrac{2\pi}{T_0}t)$;

By replacing u_C and $\ddot{u}_C$ in the differential equation : $E\cos(2\pi/T_0)t - LCE(2\pi/T_0)^2\cos(2\pi/T_0)t = 0$

$\Rightarrow 1 - LC(2\pi/T_0)^2 = 0$

$\Rightarrow T_0 = 2\pi\sqrt{LC} = 2\pi\sqrt{0.8\times10^{-6}} = 5.62$ ms. **(¾)**

3)  **(¼)**

4) The electric energy W_0 of the capacitor passes to the coil that stores it in the form of magnetic energy and vice versa. **(¼)**

C – 1) The electric energy W_0 of the capacitor passes (partially) to the coil that stores it in the form of magnetic energy and the rest of this energy is dissipated in the form of the thermal energy. **(¼)**

2) a) $3T = 17$ ms $\Rightarrow T = 5.67$ ms **(¼)**

b) T slightly greater then T_0. **(¼)**

3) a) because u_C is max. $\Rightarrow i = 0 \Rightarrow W_{mag.} = 0 \Rightarrow$ energy of the circuit = electrical energy **(¼)**

b) $W_n = 0.014\ W_0 \Rightarrow \dfrac{W_n}{W_0} = 0.014 \Rightarrow W_n = 0.014\times32\times10^{-6} = 0.448\times10^{-6}$ J. **(½)**

c) $W_n = ½ Cu_n^2 \Rightarrow u_n = 0.95$ V . Curve gives t = 17 ms = nT = n(5.62) $\Rightarrow$ n = 3 **(¾)**

Fourth exercise

A – 1) $E_m = \dfrac{1}{2}I\theta'^2 + mga(1-\cos\theta) = \dfrac{1}{2}I\theta'^2 + mga\dfrac{\theta^2}{2}$ **(¾)**

2) a) $\dfrac{dE_m}{dt} = 0 \Rightarrow I\theta'\theta'' + mga\theta\theta' = 0;\ \theta' \neq 0 \Rightarrow \theta'' + \dfrac{mga}{I}\theta = 0$ **(½)**

b) $\theta'' + \omega_0^2\theta = 0 \Rightarrow \omega_0^2 = \dfrac{mga}{I}$; $T = \dfrac{2\pi}{\omega_0} \Rightarrow T = 2\pi\sqrt{\dfrac{I}{mga}}$. **(½)**

3) a) i) $T_1 = 2\pi\sqrt{\dfrac{I_1}{mga_1}}$ and . $T_2 = 2\pi\sqrt{\dfrac{I_2}{mga_2}}$; $T_1 = T_2 \Rightarrow \dfrac{I_1}{I_2} = \dfrac{a_1}{a_2}$ **(½)**

ii) $\dfrac{I_0 + ma_1^2}{a_1} = \dfrac{I_0 + ma_2^2}{a_2} \Rightarrow I_0(a_2 - a_1) = ma_1a_2(a_2 - a_1) \Rightarrow I_0 = ma_1a_2$. **(¾)**

b) $T' = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{I_1}{mga_1}}$ $T' = 2\pi\sqrt{\dfrac{I_0 + ma_1^2}{mga_1}}$

$T' = 2\pi\sqrt{\dfrac{ma_1a_2 + ma_1^2}{mga_1}} = 2\pi\sqrt{\dfrac{a_1 + a_2}{g}} \Rightarrow \ell = a_1 + a_2$. **(1)**

B – 1) a) $a_1 = 10$ cm and $a_2 = 20$ cm **(½)**

b) $I_0 = ma_1a_2 = 4\times10^{-3}$ kgm^2 ; $\ell = a_1 + a_2 = 30$ cm. **(1)**

2) a) From graph : T_{min} for a' = 14 cm . **(½)**

b) $T = 2\pi\sqrt{\dfrac{I_0 + ma^2}{mga}}$ T is minimum when $\left(\dfrac{I_0}{mga} + \dfrac{a}{g}\right)$ is min. but

$\dfrac{I_0}{mga}\times\dfrac{a}{g} = \dfrac{I_0}{mg^2} = Cte$

thus T is min. when

$\dfrac{I_0}{mga} = \dfrac{a}{g} \Rightarrow a = \sqrt{\dfrac{I_0}{m}} = 14{,}1$ cm $\Rightarrow$ a = 14.1 cm $\Rightarrow T_{min} = 1.05$ s **(1½)**

Or $T = 2\pi\sqrt{\dfrac{I_0 + ma^2}{mga}} = \dfrac{2\pi}{\sqrt{mg}}\sqrt{\dfrac{I_0}{a} + ma} = cte \Rightarrow \dfrac{dT}{da} = 0 \Rightarrow$

$\dfrac{2\pi}{\sqrt{mg}}\dfrac{(-\dfrac{I_0}{a^2} + m)}{2\sqrt{\dfrac{I_0}{a} + ma}} = 0 \Rightarrow I_0 = ma^2$