

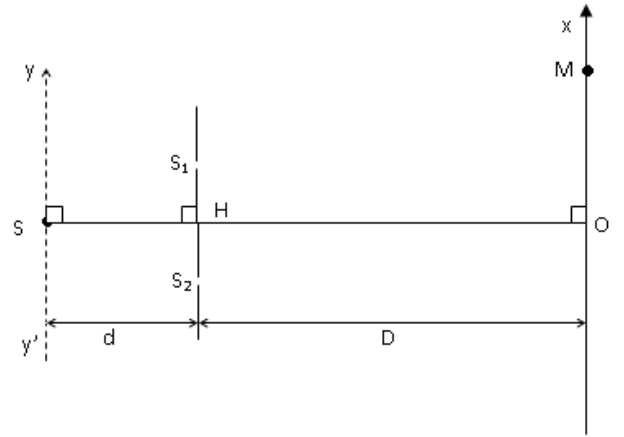
الدورة الإستثنائية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	

This exam is formed of four exercises in four pages.
The use of non-programmable calculator is allowed.

First Exercise (6 points)

Interference of Light

The object of this exercise is to show how to use Young's double slit apparatus to measure very small displacements. A source put at a point S, emitting a monochromatic radiation of wavelength λ in air, illuminates the two slits S_1 and S_2 that are separated by a distance a . The screen of observation is placed at a distance D from the plane of the slits.



1. Describe the aspect of the interference fringes observed on the screen.
2. At a point M of abscissa $x = \overline{OM}$, the optical path difference is given by the relation:

$$\delta = MS_2 - MS_1 = \frac{ax}{D}$$

- a) At the point O, we observe a bright fringe, called central bright fringe. Why?
- b) What condition must δ satisfy in order to observe, at M, a dark fringe?
- c) Give the expression of x in terms of a , D and λ , so that M is the center of a bright fringe.
- d) Given: $\lambda = 0.55 \mu\text{m}$; $a = 0.2 \text{ mm}$; $D = 1.5 \text{ m}$; $d = 10 \text{ cm}$.

We take $x = 1.65 \text{ cm}$. Are the waves interfering at M in phase or out of phase? Justify your answer.

3. We move the source from S to point S' vertically up on the axis $y'y'$ perpendicular to the horizontal axis of symmetry SO, by the distance $b = SS'$. In this case, we can write

$$S'S_2 - S'S_1 = \frac{ab}{d}$$

- a) The central bright fringe is no longer at O but at point O'.
 - i) Justify this displacement.
 - ii) Specify, with justification, the direction of this displacement.
- b) Determine the value of b , knowing that $OO' = 1 \text{ cm}$.

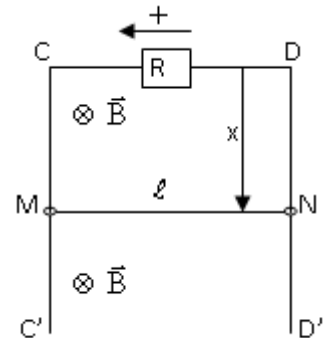
Second Exercise (8 points)

Motion of a conductor in two fields

Two vertical rails CC' and DD' are connected by a resistor of resistance R . A conducting rod MN , of mass m and length ℓ , can slide without friction along these rails and remains horizontally in contact with these rails.

The whole set-up is placed within a uniform and horizontal magnetic field \vec{B} that is perpendicular to the plane of the rails.

The rod MN , released from rest at the instant $t_0 = 0$, is found at an instant t at a distance x from CD , moving with a velocity whose algebraic value is v ($v > 0$) (adjacent figure).



1. Determine, at the instant t , the expression of the magnetic flux due to \vec{B} through the circuit CMND in terms of B , ℓ and x , taking into consideration the arbitrary positive direction as shown on the figure.
2. a) Determine the expression of:
 - i) The e.m.f "e" induced across the rod MN, in terms of v , B and ℓ .
 - ii) The induced current i in terms of R , B , ℓ and v .
- b) Indicate, with justification, the direction of the current.
3. Show that the electric power dissipated by the resistor, at the instant t , is given by : $P_{el} = \frac{B^2 \ell^2}{R} v^2$.
4. The rod MN is acted upon by two forces: its weight $m\vec{g}$ and the Laplace's force \vec{F} of magnitude $F = i \ell B$.
 - a) Applying Newton's second law, show that the differential equation in v is given by:

$$\frac{dv}{dt} + \frac{B^2 \ell^2}{mR} v = g.$$
 - b) The solution of this differential equation is: $v = A(1 - e^{-\frac{t}{\tau}})$. Show that:

$$A = \frac{mgR}{B^2 \ell^2} \text{ and } \tau = \frac{mR}{B^2 \ell^2}.$$
 - c) Show that v would attain a limiting value V_{lim} .
 - d) i) Give the expression of v as a function of V_{lim} at the instant $t = \tau$.
 - ii) Deduce the time at the end of which v attains practically its limiting value.
 - e) Calculate the value of V_{lim} and that of τ , knowing that: $\ell = 20 \text{ cm}$, $m = 10 \text{ g}$, $R = 0.1 \Omega$, $B = 0.5 \text{ T}$ and $g = 10 \text{ m/s}^2$.
5. In the steady state, starting from the instant when $v = V_{lim}$, the mechanical energy of the system (MN in the field \vec{B} , Earth) decreases.
 - a) Explain this decrease.
 - b) In what form is this energy dissipated?
 - c) Calculate the power dissipated.

Third Exercise (8 points)

Study of Charging and Discharging of a Capacitor

The adjacent circuit allows to study the variation of the voltage $u_C = u_{BM}$ across a capacitor of capacitance C during charging and discharging. We consider a generator delivering a constant voltage E , a resistor of resistance $R_1 = 25 \Omega$ and a coil of inductance L and of resistance r . Initially, the switch K is in position (0) and the capacitor is uncharged. An oscilloscope allows displaying the variation of u_C as a function of time.

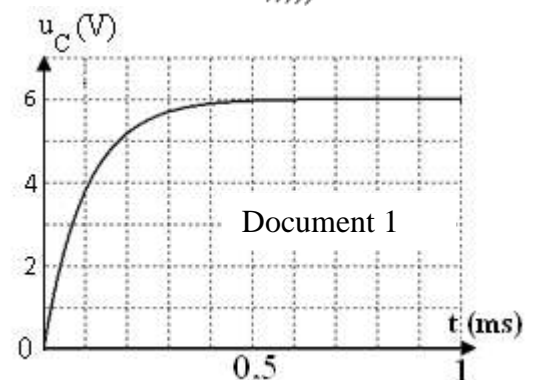
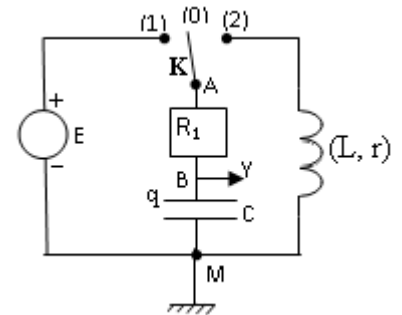
A – Charging of the capacitor

At the instant $t_0 = 0$, the switch is in position (1) and the capacitor starts charging. At an instant t , the circuit carries a current i and the capacitor carries the charge q .

1. a) Redraw the diagram of the circuit indicating on it the real direction of i .
- b) Write down the relation between i and u_C .
2. a) Derive the differential equation in u_C .
- b) The solution of this differential equation is of the form:

$$u_C = A + B e^{-\frac{t}{\tau_1}}.$$

Determine the expressions of the constants A , B and τ_1 .



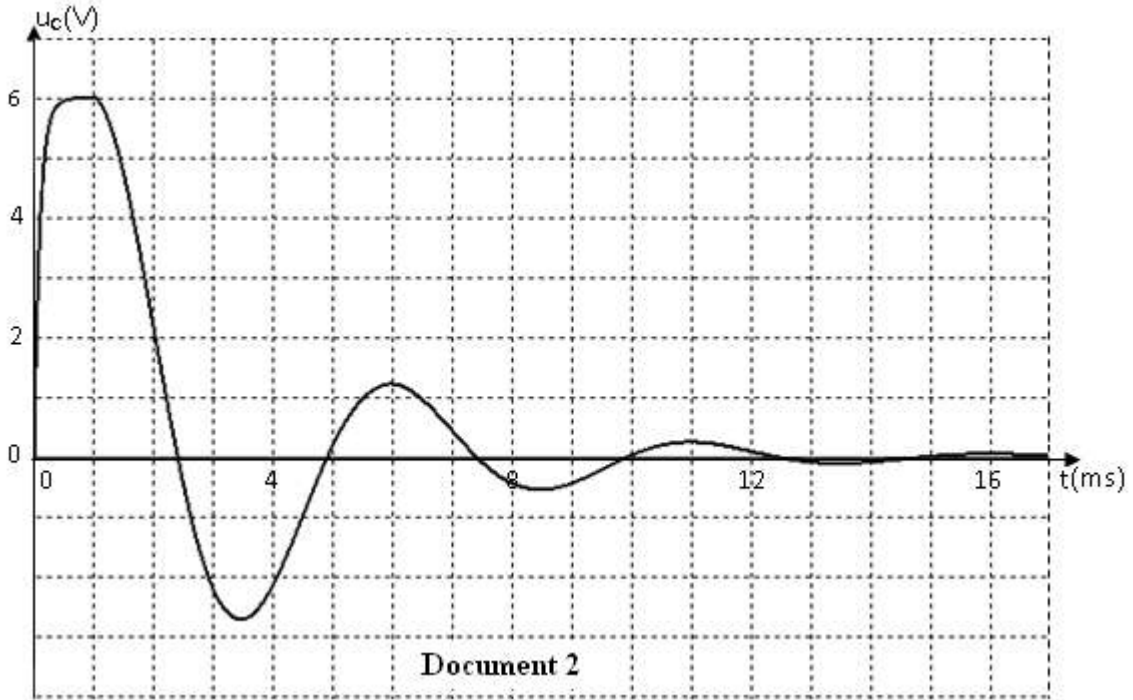
c) Referring to the graph of document 1, determine:

i) The values of E and τ_1 . Deduce that the value of C is $4\mu\text{F}$.

ii) The minimum duration at the end of which the capacitor is practically completely charged.

B – Discharging of the capacitor through a coil

K is moved from position (1) at the instant $t_1 = 0.6$ ms and becomes in position (2) at the instant $t_2 = 1$ ms. The document 2 shows the variation of u_C between the instants 0 and 17ms.



1. The voltage u_C remains constant between the instants t_1 and t_2 . Why?
2. Starting from the instant $t_2 = 1$ ms, the circuit is the seat of electric oscillations. Referring to the graph of document 2, give the value of the pseudo period T of the electric oscillations.
3. a) Write down the expression of the proper period T_0 in an LC circuit.
 b) Knowing that $L = 0.156$ H and $\left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_0}\right)^2 - \left(\frac{r + R_1}{2L}\right)^2$, calculate r .
4. a) Determine, referring to document 2, the value of u_C at the instant $t = 6$ ms.
 b) Calculate the value of the loss in electric energy in the circuit at the end of the first oscillation.

Fourth Exercise (8 points)

Mechanical Oscillators

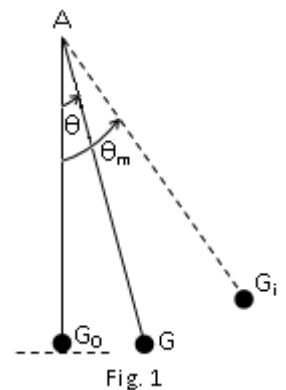
The parts A and B are independent. We neglect friction in all this exercise.

A- Simple pendulum

A simple pendulum (P) is formed of a particle G of mass m connected to one end of an inextensible string, of negligible mass and of length L ; the other extremity is connected to a fixed point A . The pendulum is shifted by an angle θ_m from its equilibrium position AG_0 to the position AG_i , then released from rest at the instant $t_0 = 0$ s; thus it oscillates with the amplitude θ_m .

At an instant t , the position of AG is defined by θ , the angular abscissa relative to its equilibrium position, and v is the algebraic measure of the velocity of G (Fig.1).

Take the horizontal plane through G_0 as a gravitational potential energy reference.

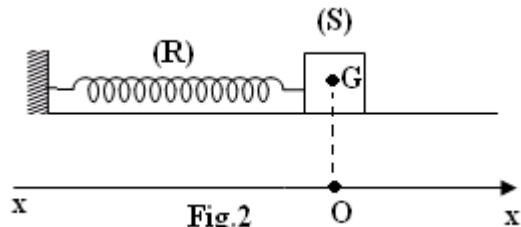


1. Find the expression of the mechanical energy of the system [(P), Earth] in terms of m , g , L , v and θ .
2. Derive the second order differential equation in θ that governs the motion of this pendulum.
3. a) What condition must θ satisfy in order to have a simple harmonic motion?
 b) Deduce, in this case, the expression of the proper period T_0 of the oscillations.
 c) Write down the time equation of motion, in the case $\theta_m = 0.1$ rad.

Take : $g = 10\text{m/s}^2$; $L = 1\text{m}$ and $\pi^2 = 10$.

B- Horizontal elastic pendulum

A solid (S) of mass m may slide on a horizontal plane; it is connected to a spring (R) of stiffness $k = 4 \text{ N/m}$. When (S) is in equilibrium, its center of mass G is found vertically above the point O, taken as origin on the horizontal axis of abscissa.



(S), shifted from its equilibrium position, is released from rest at the instant $t_0 = 0$. At an instant t , the abscissa of G is x and the algebraic value of its velocity is v .

A convenient apparatus gives the variation of x as a function of time (Fig. 3).

The horizontal plane containing G is taken as a gravitational potential energy reference.

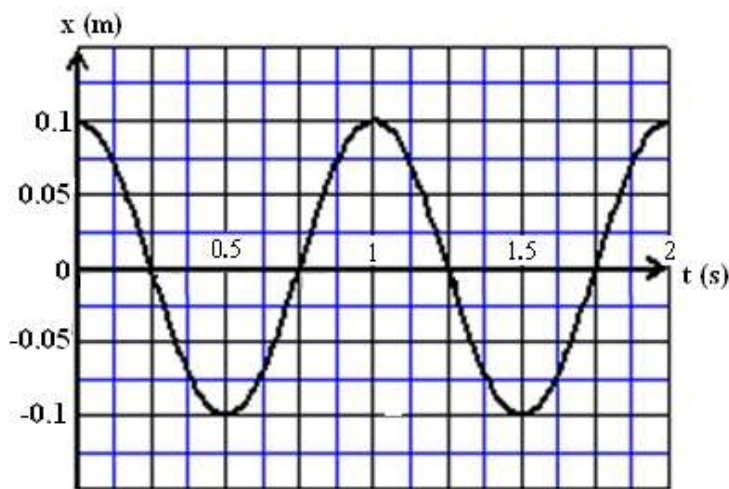


Fig. 3

1. Derive the second order differential equation in x that governs the motion of G.
2. The solution of this differential equation is of the form : $x = X_m \cos(\frac{2\pi}{T_0}t + \varphi)$, where X_m , T_0 and φ are constants.

Referring to the graph of figure (3), give the values of X_m and T_0 and determine φ .

3. a) Determine the expression of the proper period T_0 in terms of m and k .
 b) Deduce m .
4. a) Referring to the graph of figure (3), give the instants at which the elastic potential energy is maximum.
 b) Calculate then the value of the mechanical energy of the system [(S), (R), Earth].

C- Behavior of the pendulums on the Moon

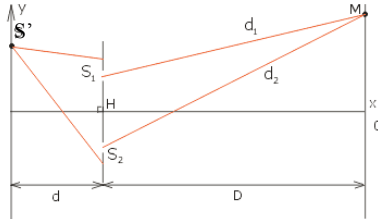
We suppose that the two preceding pendulums are now on the Moon.

Tell, with justification, for each pendulum, which of the statements in the following table is true .

Statement 1	Statement 2	Statement 3
T_0 does not vary	T_0 increases	T_0 decreases

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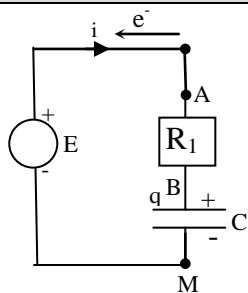
First exercise (6 points)

Part of the Q	Answer	Mark
1	Fringes are: alternating bright and dark- parallel to each other and to the slits- rectilinear- equidistant.	1/4 1/4 1/4 1/4
2.a	At point O, $\delta = 0$, all the waves arriving to O are in phase: we observe at O a bright fringe.	1/2
2.b	At point M of abscissa x, we observe a dark fringe if the path difference δ at this point is such that: $\delta = (2k+1)\frac{\lambda}{2}$ with k an integer	1/2
2.c	The abscissa x of the point M is obtained from the expression of the difference of path: $\delta = k\lambda = \frac{ax}{D} \Rightarrow x = \frac{k\lambda D}{a}$	1/2
2.d	$\delta = \frac{ax}{D} = \frac{0.2 \times 16.5}{1.5 \times 10^3} = 2.2 \times 10^{-3} \text{ mm}$. $\frac{\delta}{\lambda} = \frac{2.2 \times 10^{-3}}{0.55 \times 10^{-3}} = 4$, thus δ is a multiple integer of wavelengths: the waves that interfere at M are in phase.	1/2 1/2
3.a.i	If O remains the center of the central bright fringe : $\delta' = (S'S_2 + S_2O) - (S'S_1 + S_1O) = (S'S_2 - S'S_1) \neq 0$ But for the C.B.F we have : $\delta' = 0 \Rightarrow O$ displaced to O'.	1/2
3.a.ii	$\delta' = (S'S_2 + S_2O') - (S'S_1 + S_1O') = 0 \Rightarrow (S'S_2 - S'S_1) + (S_2O' - S_1O') = 0$ $\Rightarrow (S'S_2 - S'S_1) = - (S_2O' - S_1O')$; $(S'S_2 - S'S_1) > 0 \Rightarrow (S_2O' - S_1O') < 0 \Rightarrow S_2O' < S_1O'$ $\Rightarrow O$ displaced downward. Another meth. $\delta' = \frac{ab}{d} + \frac{ax}{D} = 0 \Rightarrow \frac{b}{d} = -\frac{x}{D} \Rightarrow x = -\frac{bD}{d} < 0 \Rightarrow$ The C.B.F displaced downward.	1/2
3.b	The central fringe on the screen corresponds to a zero path difference at this point M: $\delta' = SS_2 + S_2M - (SS_1 + S_1M)$ $\delta' = (SS_2 - SS_1) + (S_2M - S_1M)$ $\delta' = \frac{ab}{d} + \frac{ax}{D}$  The central fringe corresponds to $\delta' = 0$; its position is defined by: $\frac{ab}{d} + \frac{ax}{D} = 0$ so: $b = -\frac{d}{D}x \Rightarrow b = \frac{d}{D} x = \frac{10 \times (1)}{1.5 \times 10^2} = 0.0667 \text{ cm} = 0.667 \text{ mm}$	3/4 3/4

Second exercise (8 points)

Part of the Q	Answer	Mark
1	The magnetic flux $\phi = \vec{B} \cdot S \vec{n} = -BS = -B\ell x$.	1/4 1/4
2.a.i	The induced emf $e = -\frac{d\phi}{dt} = B\ell \frac{dx}{dt} = B\ell v$.	1/4 1/4
2.a.ii	The value of the current $i = \frac{e}{R} = \frac{B\ell v}{R}$.	1/4 1/4
2.b	$i > 0$, then i passes from M to N in the rod.	1/2
3	The dissipated electric power: $P_{el} = Ri^2 = R\left(\frac{B\ell v}{R}\right)^2 = \frac{B^2\ell^2}{R}v^2$.	1/2
4.a	$m\vec{g} + \vec{F} = \frac{d\vec{P}}{dt}$; with $F = iBL = \frac{B^2\ell^2}{R}v$ Project vertically downward: $mg - \frac{B^2\ell^2}{R}v = \frac{dP}{dt} = m\frac{dv}{dt}$. $\Rightarrow \frac{dv}{dt} + \frac{B^2\ell^2}{mR}v = g$	1/4 1/4 1/2 1/2
4.b	$\frac{dv}{dt} = \frac{A}{\tau}e^{-\frac{t}{\tau}}$; $\frac{A}{\tau}e^{-\frac{t}{\tau}} + A\frac{B^2\ell^2}{mR} - A\frac{B^2\ell^2}{mR}e^{-\frac{t}{\tau}} = g \Rightarrow A\frac{B^2\ell^2}{mR} = g$ and $\frac{A}{\tau} = A\frac{B^2\ell^2}{mR}$ $A = \frac{mgR}{B^2\ell^2}$ and $\tau = \frac{mR}{B^2\ell^2}$	1/2 1/2
4.c	When t increases, the term $e^{-\frac{t}{\tau}}$ tends towards zero and v tends towards A . Thus $V_{lim} = A = \frac{mgR}{B^2\ell^2}$.	1/2
4.d.i	For $t = \tau$, $v = V_{lim}(1 - e^{-1}) = 0.63 V_{lim}$.	1/2
4.d.ii	The conductor MN reaches practically its limiting speed for $t = 5\tau = 0.5$ s.	1/2
4.e	$V_{lim} = \frac{10^{-2} \times 10 \times 0.1}{0.5^2 \times 0.2^2} = 1$ m/s and $\tau = \frac{10^{-2} \times 0.1}{0.5^2 \times 0.2^2} = 0.1$ s	1/4 1/4
5.a	The kinetic energy does not vary since the speed of MN remains constant while the gravitational potential energy decreases	1/4
5.b	It is dissipated as Joule's effect in the resistor.	1/4
5.c	$P = \frac{B^2\ell^2}{R}v_{lim}^2 = \frac{0.5^2 \times 0.2^2}{0.1} 1^2 = 0.1$ W.	1/4 1/4

Third exercise (8 points)

Part of the Q	Answer	Mark
A.1.a	Circuit and direction of i 	1/4
A.1.b	$i = C \frac{du_C}{dt}$	1/2
A.2.a	From the law of addition of voltages: $u_{AM} = u_{AB} + u_{BM} \Rightarrow E = R_1 i + u_C$ $\Rightarrow R_1 C \frac{du_C}{dt} + u_C = E$	1/2 1/2
A.2.b	$u_C = A + B e^{-\frac{t}{\tau_1}}$ At $t = 0 \Rightarrow u_C = 0 \Rightarrow A + B = 0 \Rightarrow A = -B \Rightarrow u_C = A - A e^{-\frac{t}{\tau_1}}$ $\frac{du_C}{dt} = \frac{A}{\tau_1} e^{-\frac{t}{\tau_1}} \Rightarrow A \frac{R_1 C}{\tau_1} e^{-\frac{t}{\tau_1}} + A - A e^{-\frac{t}{\tau_1}} = E;$ $\Rightarrow A = E$ and $\frac{R_1 C}{\tau_1} - 1 = 0 \Rightarrow \tau_1 = R_1 C.$	1/2 1/2 1/2
A.2.c.i	$(u_C)_\ell = E = 6 \text{ V}.$ For $t = \tau_1, u_C = 0.63 E = 3.78 \text{ V}.$ From the graph: $\tau_1 = 0.1 \text{ ms}.$ $\tau_1 = R_1 C \Rightarrow C = 4 \times 10^{-6} \text{ F}.$	1/4 1/2 1/4
A.2.c.ii	$t_{\min} = 5\tau_1 = 0.5 \text{ ms} = 5 \times 10^{-4} \text{ s}.$	1/4 1/4
B.1	During the passage from (1) to (2), the circuit is open and the voltage u_C does not vary and retains the value of 6 V between $t_1 = 0.6 \text{ ms}$ and $t_2 = 1 \text{ ms}$	1/2
B.2	$T = 5 \text{ ms} = 5 \times 10^{-3} \text{ s}$	1/2
B.3.a	$T_0 = 2\pi \sqrt{LC}.$	1/4
B.3.b	$T_0 = 2\pi \sqrt{LC} \approx 4.96 \times 10^{-3} \text{ s},$ replace in the given equation to get $r \approx 23 \Omega$	1/4 1/2
B.4.a	At the instant $t = 6 \text{ ms}, u_C = 1.25 \text{ V}$	1/2
B.4.b	$W = W_0 - W_1 = \frac{1}{2} C (E^2 - u_C^2) = \frac{1}{2} \times 4 \times 10^{-6} (36 - 1.25^2) = 6.89 \times 10^{-5} \text{ J}$	1/2 1/4

Fourth exercise (8 points)

Part of the Q	Answer	Mark
A.1	$ME = KE_k + PE_g = \frac{1}{2} mv^2 + mgL(1 - \cos\theta)$.	$\frac{1}{4}$ $\frac{1}{2}$
A.2	$\frac{dME}{dt} = 0 = mvx'' + mgL\theta g \sin\theta = mL^2\theta'' + mgL\theta' \sin\theta \Rightarrow \theta'' + \frac{g}{L} \sin\theta = 0$.	$\frac{1}{2}$
A.3.a	θ_m should be $< 10^\circ$. $\forall t$.	$\frac{1}{4}$
A.3.b	$\theta_m < 10^\circ \Rightarrow \theta < 10^\circ \Rightarrow \sin\theta = \theta$ in rad ; in this case: the differential equation is: $\theta'' + \frac{g}{L} \theta = 0$ \Rightarrow The proper angular frequency ω_0 is such that $(\omega_0)^2 = \frac{g}{L} \Rightarrow \omega_0 = \sqrt{\frac{g}{L}}$. The proper period is $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{L}{g}}$.	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$
A.3.c	Let $\theta = \theta_m \sin(\omega_0 t + \varphi)$; with $\theta_m = 0.1$ rad and $\omega_0 = \sqrt{\frac{g}{L}} = \pi$ rad/s ; at $t = 0$ we have: $\theta = \theta_m \sin\varphi = \theta_m \Rightarrow \sin\varphi = 1$ $\Rightarrow \varphi = \frac{\pi}{2}$ rad $\Rightarrow \theta = 0.1 \sin(\pi t + \frac{\pi}{2})$	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
B.1	$ME = KE + PE_{elastic} + PE_g = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 + 0 = \text{const.}$ $\frac{dME}{dt} = 0 = mvx'' + kxv \Rightarrow x'' + \frac{k}{m} x = 0$.	$\frac{1}{4}$ $\frac{1}{4}$
B.2	$X_m = 0.1$ m ; $T_0 = 1$ s ; if $t = 0$, $x = X_m \cos\varphi = X_m \Rightarrow \cos\varphi = 1 \Rightarrow \varphi = 0$.	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
B.3.a	$x = X_m \cos(\frac{2\pi}{T_0} t + \varphi) \Rightarrow v = -X_m \frac{2\pi}{T_0} \sin(\frac{2\pi}{T_0} t + \varphi)$ $\Rightarrow x'' = -X_m \frac{2\pi}{T_0} \frac{2\pi}{T_0} \cos(\frac{2\pi}{T_0} t + \varphi)$; replacing in the differential equation, we obtain: $-X_m \frac{2\pi}{T_0} \frac{2\pi}{T_0} \cos(\frac{2\pi}{T_0} t + \varphi) + \frac{k}{m} X_m \cos(\frac{2\pi}{T_0} t + \varphi) = 0 \Rightarrow \frac{2\pi}{T_0} = \sqrt{\frac{k}{m}} \Rightarrow$ $T_0 = 2\pi\sqrt{\frac{m}{k}}$.	1
B.3.b	$T_0 = 1$ s and $k = 4$ N/kg we obtain: $m = 0.1$ kg	$\frac{1}{2}$
B.4.a	$E_{pe} = \frac{1}{2} kx^2$, E_{pe} is max. if $ x $ is max. $\Rightarrow x = \pm 0.1$ m \Rightarrow at the instants 0, 0,5s, 1s, 1,5s, 2s .	$\frac{1}{4}$ $\frac{1}{4}$
B.4.b	For $P.E_g$ max, $KE = 0 \Rightarrow ME = P.E_g = \frac{1}{2} k(X_m)^2 = 2 \times 10^{-2}$ J.	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
C	For the simple pendulum $T_0 = 2\pi\sqrt{\frac{L}{g}}$; but $g_{moon} < g_{earth} = g$ so T_0 increases. Statement (2) For the elastic pendulum $T_0 = 2\pi\sqrt{\frac{m}{k}}$; T_0 does not vary on the Moon.(Statement 1)	$\frac{1}{2}$ $\frac{1}{4}$