الدورة الإستثنائية للعام 2012	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	

<u>This exam is formed of four exercises in four pages numbered from 1 to 4.</u>
<u>The use of a non-programmable calculator is recommended.</u>

<u>First exercise</u>: (7 ½ points)

Mechanical oscillator

A metallic rigid rod MN, of mass m = 0.25 kg, can slide without friction, on two parallel and horizontal metallic rails PP' and QQ'. During sliding, the rod remains perpendicular to the rails. These two rails, separated by a distance ℓ , are connected by a resistor of resistance R (Fig.1).
Neglect the resistance of the rod and of the rails.

A – Electromagnetic induction

The whole setup is placed in an upward, uniform and vertical magnetic field $\vec{B}$ of value B. The position of G, the center of inertia of the rod, is defined by its abscissa x on the horizontal axis (O, $\vec{i}$) with O corresponding to the position of G at $t_0 = 0$. Let O'O = d.
At an instant t, G has an abscissa $\overline{OG} = x$ and a velocity $\vec{v}$ of algebraic value v (Fig. 1).

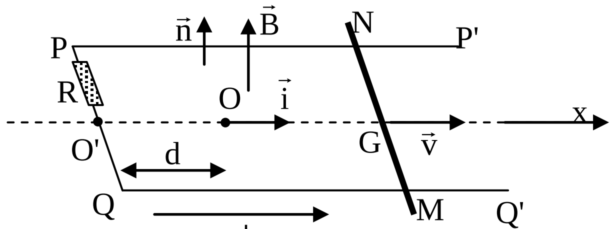

Fig. 1

1) Show that, taking into consideration the arbitrary positive direction chosen in figure 1, the expression of the magnetic flux crossing the surface limited by the circuit MNPQ is given by: $\varphi = B(d+x)\ell$.

2) a) Derive the expression of the induced e.m.f. "e" across the terminals of the rod MN in terms of B, ℓ and v.

b) i) Derive the expression of the induced current i in the circuit in terms of R, ℓ, B and v.
ii) Deduce the direction of the induced current.

3) Show that the expression of the electromagnetic force $\vec{F}$ acting on the rod can be written as:
$$\vec{F} = \frac{-B^2\ell^2}{R}\vec{v}.$$

B – Free un-damped oscillations

We remove the magnetic field $\vec{B}$.
The center of inertia G of the rod is attached to a horizontal massless spring, of un-stretched length L_0 = O'O = d and stiffness k = 50 N/m. Thus at equilibrium the abscissa of G is x = 0.
The rod, is displaced by a distance X_m = 10 cm in the positive direction, and is then released without initial velocity at the instant $t_0 = 0$; the rod thus oscillates around its equilibrium position. At an instant t, G has an abscissa x and a velocity $\vec{v}$ of algebraic value v (Fig. 2).

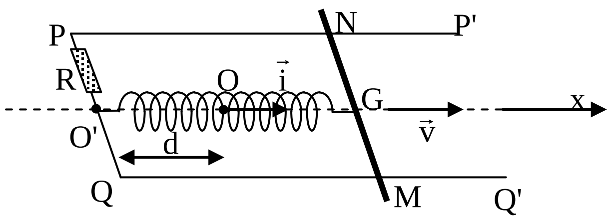

Fig. 2

1) Write, in terms of m, v, k and x, the expression of the mechanical energy of the system (rod, spring, Earth).
Take the horizontal plane through G as a gravitational potential energy reference.

2) Derive the differential equation of the second order in x, which describes the motion of G.

3) The solution of this differential equation is of the form: $x = A\cos(\omega t + \phi)$. Determine the values of the constants ω, A and ϕ (A > 0).

C – Free damped oscillations

The setup of figure 2 is placed now in the magnetic field $\vec{B}$. The rod is displaced again by X_m = 10 cm in the positive direction, and is then released without initial velocity at the instant $t_0 = 0$; the rod thus oscillates around its equilibrium position. At an instant t, G has an abscissa x and a velocity $\vec{v}$ of algebraic value v (Fig. 3).

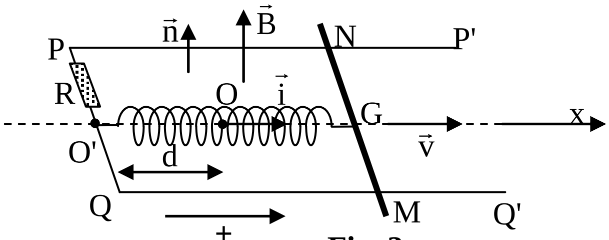

Fig. 3

1) Calculate, at the instant $t_0 = 0$, the mechanical energy of the system (rod, spring, Earth). Take the horizontal plane through G as a gravitational potential energy reference.
2) During its motion, the oscillator loses mechanical energy.
   a) Show that the power of the electromagnetic force $\vec{F}$, exerted on the rod, is given by: $P = \frac{-B^2\ell^2v^2}{R}$.
   b) Determine the expression of the power lost due to Joule's effect in the resistor in terms of B , ℓ, v and R.
   c) Deduce in what form the energy of the oscillator is dissipated.
3) After a few oscillations, the rod stops. Give , in J, the value of the total energy dissipated by the oscillator during its motion.

Second exercise: (7 ½ points)

Determination of the characteristics of a coil

In order to determine the inductance L and the resistance r of a coil, we connect the coil in series with a capacitor of capacitance C = 160 μF across the terminals of a low frequency generator (LFG) delivering an alternating sinusoidal voltage: $u_g = u_{AD} = 20\sin(100\pi t)$; (u_g in V , t in s).
The circuit thus carries an alternating sinusoidal current i.
An oscilloscope is connected so as to display the voltage $u_g = u_{AD}$ on the channel Y_A and the voltage $u_C = u_{BD}$ on the channel Y_B (Fig. 1).
On the screen of the oscilloscope we observe a display of the waveforms represented in figure 2. Take $\pi = \frac{1}{0.32}$.

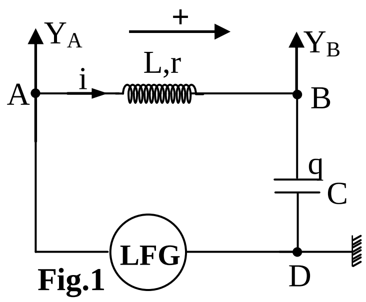

Fig.1

1) Knowing that the vertical sensitivity S_V is the same on both channels, calculate its value.
2) Calculate the phase difference between u_g and u_C. Which of them lags behind the other?
3) Deduce the expression of the voltage u_C across the terminals of the capacitor as a function of time.
4) Using the relation between the current i and the voltage u_C, determine the expression of i as a function of time.
5) Applying the law of addition of voltages, and by giving the time t two particular values, determine r and L.
6) In order to verify the preceding calculated values of L and r, we proceed as follows:
   - we measure the average power consumed in the circuit for ω =100π rad/s and we obtain 8.66 W.
   - we keep the maximum value of u_g constant but we vary its frequency f; for f = 71 Hz the effective value of the current in the circuit is maximum .

   Determine the values of r and L.

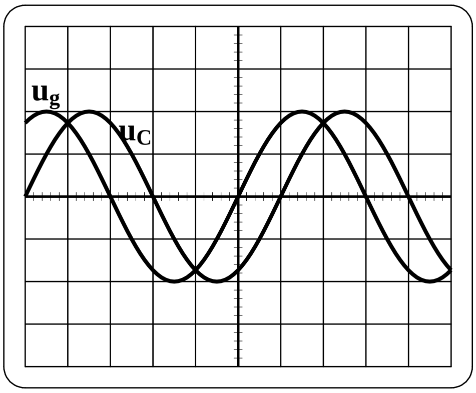

Fig. 2

Third exercise: (7 ½ points)

Diffraction and interference of light

A laser source emits a monochromatic cylindrical beam of light of wavelength λ = 640 nm in air.

A – Diffraction

This beam falls normally on a vertical screen (P) having a horizontal slit F_1 of width a. The phenomenon of diffraction is observed on a screen (E) parallel to (P) and situated at a distance D = 4 m from (P).

Consider on (E) a point M so that M coincides with the second dark fringe counted from O, the center of the central bright fringe. OIM = θ (θ is very small) is the angle of diffraction corresponding to the second dark fringe (Fig. 1).

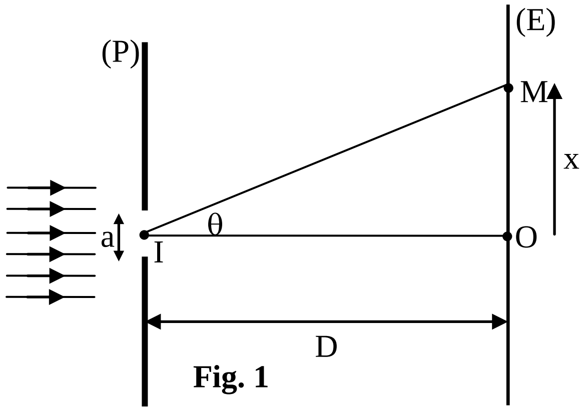

Fig. 1

1) Write the expression of θ in terms of a and λ.
2) Determine the expression of OM = x in terms of a, D and λ.
3) Determine the value of a if OM = 1.28 cm.
4) We replace the slit F_1 by another slit F_1' of width 100 times larger than that of F_1. What do we observe on the screen (E)?

B – Interference

We cut in (P) another slit F_2 identical and parallel to F_1 so that the distance F_1F_2 = a' = 1 mm. The laser beam falls normally on the two slits F_1 and F_2. We observe on (E) a system of interference fringes. O' is the orthogonal projection of the midpoint I' of F_1F_2 on (E) (Fig. 2).

1) **a)** Due to what is the phenomenon of interference?
   b) Describe the fringes observed on (E).
2) Consider a point N on (E) so that O'N = x.
   a) Write the expression of the optical path difference $\delta = F_2N - F_1N$ in terms of a', x and D.
   b) If N is the center of a dark fringe of order k, write the expression of the optical path difference δ at N in terms of λ and k.
   c) Deduce the expression of x in terms of a', D, k and λ.
   d) Knowing that the interfringe distance i is the distance between the centers of two consecutive dark fringes, deduce then the expression of i in terms of λ, D and a'.

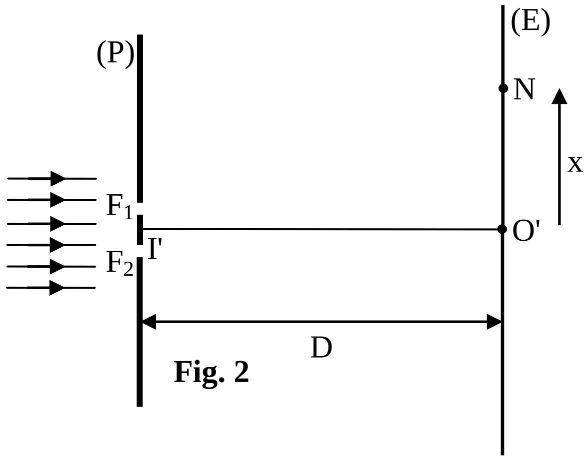

Fig. 2

3) The whole set up of figure 2 is immersed in water of index of refraction n.
   a) **i)** The interfringe distance i varies and becomes i'. Why?
   ii) Show that $i' = \frac{i}{n}$.
   b) We move (E) parallel to (P) and away from it by a distance d = $\frac{4}{3}$ m. We notice that the interfringe distance takes again the initial value i.
   Deduce then the value of n.

Fourth exercise: (7 ½ points)

Nuclear reactor

Given:

- Atomic mass unit 1u = 931.5 MeV/c^2 = 1.66×10^{-27} kg;
- 1 MeV = 1.6×10^{-13} J;
- Mass of particles (in u): antineutrino $^{0}_{0}\bar{\nu} \approx 0$; electron $^{0}_{-1}e$: 5.5×10^{-4} ; neutron $^{1}_{0}n$: 1.0087.

Element	**Molybdenum**		**Technetium**		**Tellurium**
Nuclide	$^{101}_{42}Mo$	$^{102}_{42}Mo$	$^{101}_{43}Tc$	$^{102}_{43}Tc$	$^{135}_{52}Te$
Mass (in u)	100.9073	101.9103	100.9073	101.9092	134.9167

Element	**Uranium**		**Neptunium**	**Plutonium**
Nuclide	$^{235}_{92}U$	$^{238}_{92}U$	$^{239}_{93}Np$	$^{239}_{94}Pu$
Mass (in u)	235.0439	238.0508	239.0533	239.0530

Read carefully the following text about fast neutrons, and answer the questions that follow.

" …the basic substance used to obtain nuclear energy is the natural uranium which is mainly formed of the two isotopes: uranium 235 and uranium 238…

… The fast-neutron nuclear reactors (breeder reactors), use uranium 235 or plutonium 239 (or the two at the same time) as fuel. In each reactor we put around the core which is constituted of uranium 235 ($^{235}_{92}U$), a cover made essentially of fertile uranium 238 ($^{238}_{92}U$). This cover can trap fast neutrons issued from the fission reactions of uranium 235.

These reactors transform more uranium 238 atoms into plutonium 239.

Finally, in the very well studied fast neutrons reactors, the quantity of fissionable matter that is created, exceeds notably the consumed quantity. For this reason, these reactors are called breeder reactors…"

Questions

1) **a)** What is meant by isotopes of an element?

b) Give the composition of each of uranium 235 and uranium 238 nuclei.

2) In the reactor, the uranium 238 reacts with the fast neutrons according to the reaction:

$^{238}_{92}U + ^{1}_{0}n \longrightarrow X$ (reaction 1)

The obtained nucleus X is radioactive and after two successive β^- emissions , it is transformed into plutonium:

$X \longrightarrow ^{0}_{-1}e + Y + ^{0}_{0}\bar{\nu}$ (reaction 2)

$Y \longrightarrow ^{0}_{-1}e + _{94}Pu + ^{0}_{0}\bar{\nu}$ (reaction 3)

a) Identify X and Y.

b) Deduce the nuclear reaction that occurs between a $^{238}_{92}U$ nucleus and a fast neutron that leads to the formation of a plutonium 239 (reaction 4).

3) The plutonium 239 ($^{239}_{94}Pu$) is fissile and can react with neutrons according to the reaction :

$^{239}_{94}Pu + ^{1}_{0}n \longrightarrow B + ^{135}_{52}Te + 3\,^{1}_{0}n$ (reaction 5)

a) Identify B.

b) Calculate, in MeV/c^2, the mass defect Δm in reaction 5.

c) Deduce, in Mev, the energy E liberated during the fission of a plutonium nucleus.

d) Find, in joules, the energy liberated by the fission of one kilogram of plutonium.

4) From reactions 4 and 5, justify the definition of a breeder reactor given in the text.

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مشروع معيار التصحيح	مسابقة في مادة الفيزياء المدة ثلاث ساعات	**الاسم:** **الرقم:**

First exercise: Mechanical oscillator		**7 ½**
Part of the Q	**Answer**	**Mark**
A.1	The flux $\varphi = S\vec{B}.\vec{n} = BS\cos(\theta) = B(d+x)\ell$. [$\theta = 0$]	½
A.2.a	The induced emf "e" : $e = -\frac{d\varphi}{dt} = -B\ell v$;	½
A.2.b.i	The value i of the induced electric current: $i = \frac{e}{R} = -\frac{B\ell v}{R}$	½
A.2.b.ii	$i < 0$, then the direction of i is opposite to the chosen positive direction, it is directed from N to M in the rod.	¼
A.3	The direction of Laplace's force is opposite to that of the motion (From Lenz's law: since the electromagnetic effect of the induced current opposes the causes of it). $F = i\ell B, = (\frac{-B\ell v}{R}).B\ell = \frac{-B^2\ell^2 v}{R} \Rightarrow \vec{F} = \frac{-B^2\ell^2}{R}\vec{v}$.	**1**
B.1	$ME = ½ mv^2 + ½ kx^2$.	½
B.2	No friction, conservation of mechanical energy, ME = constant. Derive ME with respect to time: $\frac{dME}{dt} = 0$; $mvv' + kxx' = 0 \Rightarrow mx'' + kx = 0$ ($v = x'$ and $v' = x''$) $\Rightarrow x'' + \frac{k}{m}x = 0$.	¾
B.3	$x' = -\omega A\sin(\omega t + \varphi)$ and $x'' = -\omega_0^2 A\cos(\omega t + \varphi)$. By replacing in the differential equation: $\Rightarrow \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} = 14.1$ rd/s For $t_0 = 0$, $x' = 0 = \omega A\sin(\varphi) = 0 \Rightarrow \varphi = 0$ or π. Also: for $t_0 = 0$, $x = X_m = A\cos(\varphi) > 0$ and $A > 0$, then $\varphi = 0$ and A = 10 cm.	**1 ½**
C.1	The initial mechanical energy is: $ME_o = ½ k x_m^2 = ½ \times 50 \times 0.01 = 0.25$ J.	½
C.2.a	The power of the force $\vec{F}$ is : $\vec{F}\cdot\vec{v} = P = \frac{-B^2\ell^2 v^2}{R}$.	¼
C.2.b	The power lost by Joule's effect is: $Ri^2 = R\times\left(\frac{B\ell v}{R}\right)^2 = \frac{B^2\ell^2 v^2}{R}$	½
C.2.c	Since $\lvert P_{electromagnetic}\rvert = \lvert P_{thermal}\rvert$ therefore the mechanical energy is totally transformed into thermal energy in the resistor.	½
C.3	The total dissipated energy: $= ME_o = 0.25$ J.	¼

Second exercise: Determination of the characteristics of a coil		7 ½
Part of the Q	**Answer**	**Mark**
1	The maximum voltage across the terminals of the generator corresponds to 2 div $\Rightarrow$ $S_V = \frac{20}{2} = 10V/div$	½
2	The phase difference extends over 1 div and the period over 6 div $\varphi = \lvert\varphi\rvert = \frac{1\times 2\pi}{6} = \frac{\pi}{3}$ rad ; u_C lags u_g	**1**
3	$U_C)_{max} = 20V$, $\omega = 100\pi$ and u_C lags u_g $\Rightarrow u_C = 20\sin(100\pi t - \frac{\pi}{3})$	**1**
4	$i = C\frac{du_c}{dt}$; $\frac{du_C}{dt} = 2\times 10^3\pi\cos(100\pi t - \frac{\pi}{3})$ $\Rightarrow i = 160\times 10^{-6}\times 2\times 10^3\pi\times\cos(100\pi t - \frac{\pi}{3}) = \cos(100\pi t - \frac{\pi}{3})$	**1 ½**
5	$u_{AD} = u_{AB} + u_{BD}$. $20\sin(100\pi t) = ri + L\frac{di}{dt} + u_C$ $20\sin(100\pi t) = r\cos(100\pi t - \frac{\pi}{3}) - 100\pi L\sin(100\pi t - \frac{\pi}{3}) + 20\sin(100\pi t - \frac{\pi}{3})$ * For $100\pi t = \frac{\pi}{3}$ we obtaim : $20\frac{\sqrt{3}}{2} = r \Rightarrow r = 10\sqrt{3}\,\Omega$ * For t = 0 we obtain : $L = \frac{1}{10\pi} = 0.032H$	**2**
6	The electric power is consumed only in the resistor of the coil : $P = r(I_{eff})^2 = 8.66 = \left(\frac{1}{\sqrt{2}}\right)^2 r \Rightarrow r = 17.3\Omega$ The observed phenomenon is the current resonance. In this case we have : $f = \frac{1}{2\pi\sqrt{Lc}} = 71 \Rightarrow L = 0.03$ H.	**1 ½**

Third exercise: Diffraction and interference of light		7 ½
Part of the Q	**Answer**	**Mark**
A.1	M is a dark fringe if $\sin\theta = n\frac{\lambda}{a} = \theta$, the second fringe: n =2 then $\theta = 2\frac{\lambda}{a}$	**1**
A.2	$\tan\theta = \theta = \frac{OM}{D} = \frac{x}{D}$ then $\mathbf{x} = OM = D\times\theta = \frac{2D\lambda}{a}$	¾
A.3	$a = \frac{2\lambda D}{x} = 0.4mm$	¾
A.4	We observe a spot of light.	½
B.1.a	It is due to the superposition of 2 luminous radiations.	½
B.1.b	bright and dark fringes. parallel , rectilinear and equidistant	½
B.2.a	$\delta = \frac{a'x}{D}$	¼
B.2.b	$\delta = (2k+1)\frac{\lambda}{2}$	¼
B.2.c	$x = (2k+1)\frac{\lambda D}{2a'}$	½
B.2.d	$i = x_{k+1} - x_k = \frac{\lambda D}{a'}$	½
B.3.a.i	$i = \frac{\lambda D}{a}$ and λ varies because the speed of light varies, $\Rightarrow$ i varies.	½
B.3.a.ii	$\lambda' = \frac{v}{\nu}$ and $v = \frac{c}{n} \Rightarrow \lambda' = \frac{\lambda}{n}$; $i' = \frac{\lambda' D}{a'} = \frac{\lambda D}{na'} = \frac{i}{n}$	¾
B.3.b	D' = D + d ; $i = \frac{\lambda(D+d)}{na'} = \frac{\lambda D}{a'} \Rightarrow \frac{(D+d)}{n} = D \Rightarrow n = \frac{D+d}{D} = 1.33$	¾

Fourth exercise: Nuclear reactor		7 ½
Part of the Q	**Answer**	**Mark**
1.a	We call isotopes the nuclei that have the same atomic number Z and of different mass numbers A.	½
1.b	Uranium nuclei have 92 protons and 143 neutrons for $^{235}_{92}U$, 146 neutrons for $^{238}_{92}U$.	½
2.a	$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}X$ (reaction 1); $^{239}_{92}X$ is an isotope of uranium. $^{239}_{92}U \rightarrow ^{0}_{-1}e + ^{239}_{93}Y + ^{0}_{0}\bar{\nu}$ (reaction 2); $^{239}_{93}Y$ is a nucleus of neptunium $^{239}_{93}Np$. $^{239}_{93}Np \rightarrow ^{0}_{-1}e + ^{239}_{94}Pu + ^{0}_{0}\bar{\nu}$ (reaction 3)	1 ½
2.b	The addition of (1) + (2) + (3) gives the nuclear reaction leading to the formation of plutonium : $^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{94}Pu + 2\,^{0}_{-1}e + 2\,^{0}_{0}\bar{\nu}$. (reaction 4)	1
3.a	$^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{102}_{42}B + ^{135}_{52}Te + 3\,^{1}_{0}n$ (reaction 5) $^{102}_{42}B$ is a nucleus of molybdenum $^{102}_{42}Mo$	½
3.b	$\Delta m = 239.053 + 1.0087 - (101.9103 + 134.9167 + 3\times1.0087)$ $= 0.2086\ u = 0.2086 \times 931.5\ MeV/c^2 = 194.3\ MeV/c^2$.	1
3.c	$E = \Delta m \times c^2 = 194.3\ MeV$	½
3.d	The number of plutonium nuclei in 1 kg is: $N = \dfrac{1}{1.66\times10^{-27}\times239.053} = 2.52\times10^{24}$ nuclei $E' = 2.52\times10^{24}\times194.3\times1.6\times10^{-13} = 7.83\times10^{13}$ J.	1 ½
4	Under the action of an incident neutron, a plutonium nucleus reacts according to the equation (5) and liberates three neutrons during its fission. For these three neutrons: ❖ one is used to sustain the fission reaction of plutonium; ❖ the two others are available to react with the uranium 238 to create two new plutonium atoms. For one fissile plutonium nucleus consumed, two fissile plutonium nuclei are created. This justifies the appelation of breeder reactor giving to such reactor.	½