مسابقة في مادة الفيزياء	الاسم:
المدة ثلاث ساعات	الرقم:

This exam is formed of four exercises in four pages numbered from 1 to 4.
The use of non-programmable calculator is recommended.

First exercise: (7 ½ points) **Measure of a small displacement**

The aim of this exercise is to measure a very small displacement of an apparatus.

In order to do that we attach to the apparatus a monochromatic source (S) of wavelength λ in vacuum.

Given : Planck's constant : $h = 6.62\times10^{-34}$ J.s ; $c = 3\times10^{8}$ m/s ; 1 eV = 1.6×10^{-19} J ; mass of an electron: $m_e = 9.1\times10^{-31}$ kg.

A – Determination of the wavelength λ

The source (S) illuminates the cathode of a photo cell which is covered by cesium whose work function is $W_0 = 1.9$ eV.

1) Calculate the threshold wavelength λ_o of cesium.

2) The maximum speed of an emitted photoelectron from the cathode is 2.37×10^{5} m/s.

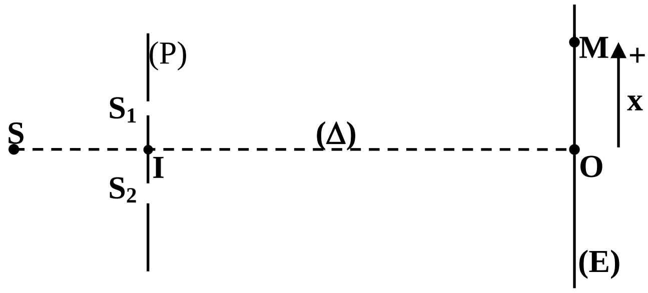
a) Calculate the maximum kinetic energy of the emitted photoelectrons.

b) Deduce that the value of the wavelength of the incident light is $\lambda = 0.602$ μm.

B – Determination of the displacement of an apparatus

The source (S), of wavelength λ, is placed on the axis of symmetry (Δ) of two small and parallel slits S_1 and S_2, separated by a distance a = 0.8 mm and made in an opaque screen (P). A screen (E) is placed parallel to (P) and at distance D = 1.6 m. I and O are the intersection of (P) and (E) with the axis (Δ) respectively (adjacent figure).

(S) illuminates the two slits S_1 and S_2.



1) Redraw the figure and show on it the region of interference.

2) On the screen (E), we observe a set of fringes. Point M is found in the region of interference on the screen (E) and is defined by its abscissa $x = \overline{OM}$.

a) Describe the interference fringes observed on the screen.

b) Express, in terms of x, D and a, the optical path difference $\delta = S_2M - S_1M$.

c) Deduce, in terms of λ, D and a, the expression of the abscissa x of M such that M is the center of:

i. a bright fringe;

ii. a dark fringe.

d) Show that O is the center of the central fringe.

3) **a)** Determine, in terms of λ, D and a, the expression of the interfringe distance i. Calculate its value.

b) Specify the nature and the order of the fringe whose center has an abscissa is x = – 4.2 mm.

4) When we displace (S) towards the two slits and along the axis (Δ). Does the position of the central fringe change? Justify.

5) The source (S) is maintained on the axis (Δ) at a distance d = 8 mm from I. We displace (S) slowly and perpendicularly to (Δ) towards one of the two slits. The optical path difference δ' at M is given by:

$\delta' = \dfrac{ax}{D} + \dfrac{ay}{d}$; where "y" is the displacement of (S). Knowing that the new position of the central fringe is the position that was originally occupied by the center of the bright fringe of order + 1 before the displacement of (S), determine "y" and deduce the direction of the displacement of (S).

Second exercise: (7 ½ points) **Nuclear power plant**

A – Nuclear fission reaction

A nuclear reactor uses enriched uranium constituted of 3% of ${}^{235}_{92}U$ and 97 % of ${}^{238}_{92}U$.

1) One of the possible nuclear fission reaction of uranium 235 is:

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{94}_{...}Sr + {}^{140}_{54}Xe + ...{}^{1}_{0}n \qquad (1) $$

a) Define the term fissile isotope.

b) Complete the equation of the reaction and specify the used laws.

2) The binding energy per nucleon of the nuclei of reaction (1) are given in the table below:

nucleus	${}^{235}_{92}U$	${}^{140}_{54}Xe$	${}^{94}_{...}Sr$
Binding energy per nucleon $\frac{E_B}{A}$	7.5 MeV	8.2 MeV	8.5 MeV

Calculate the binding energy E_B of each nucleus.

3) **a)** Determine the expression of the mass of a nucleus ${}^{A}_{Z}X$ in terms of A, Z, m_P (mass of proton), m_n (mass of neutron), E_B and c (speed of light in vacuum).

b) Show that the liberated energy by reaction (1) can be written as: $E_{lib} = E_B(Sr) + E_B(Xe) - E_B(U)$.

c) Calculate this energy in MeV.

4) In the core of the reactor, the fission of one uranium 235 nucleus liberates on average an energy of 200 MeV. 30% of this energy is transformed into electrical energy. A power plant furnishes an electric power of 1350 MW. Determine, in kg, the daily consumption of ${}^{235}_{92}U$ in this power plant.

Given: 1MeV = 1.6×10^{-13} J; $N_A = 6.02\times10^{23}$ mol^{-1} ; molar mass of ${}^{235}U$ = 235g.

B – Danger of radioactivity

Iodine131 is one of the emitted gases from a nuclear reactor. ${}^{131}_{53}I$ is a β^- emitter of half – life T = 8 days and the daughter nucleus is (Tellurium)Te.

1) **a)** The disintegration of ${}^{131}_{53}I$ nucleus is usually accompanied with the emission of γ.
Write the equation of disintegration of iodine 131.

b) Indicate the cause of the emission of γ .

2) The iodine 131 causes serious problems because it has the ability to be fixed on the thyroid gland. Let A_0 be the activity of a sample of iodine 131 at an instant $t_0 = 0$ and A is its activity at an instant t.

a) Calculate, in day^{-1}, the value of the decay constant λ of iodine 131.

b) Determine the expression of $-\ell n\left(\frac{A}{A_0}\right)$ in terms of λ and t.

c) Trace, between t = 0 and t = 32 days, the curve that represents $-\ell n\left(\frac{A}{A_0}\right)$ as a function of t.
Take the scale: on the abscissa 1 cm ↔ 4 days ; on the ordinate 1 cm ↔ 0.5.

d) we suppose that the effect of iodine on organism becomes approximately negligible when its activity becomes one-tenth of its initial activity. Determine from the traced curve the time at which there is no effect on organisms.

<u>**Third exercise**</u>**: (7 ½ points)** **Capacitor and coil**

The aim of this exercise is to determine, by different methods, the characteristics of a capacitor and a coil.

A – RC circuit

Consider a series circuit formed, of a resistor of resistance R = 100 Ω, a neutral capacitor of capacitance C and a switch K, fed by a generator of negligible internal resistance and of emf E (Fig.1).

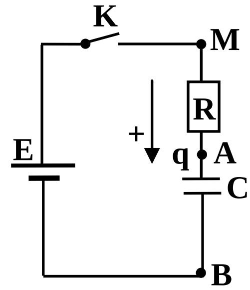


Fig.1

At the instant $t_0 = 0$, we close the switch K; then a current i flows in the circuit.

1) Derive the differential equation that describes the variation of $u_C = u_{AB}$ as a function of time.

2) The solution of this differential equation is $u_C = E(1 - e^{\frac{-t}{\tau}})$.

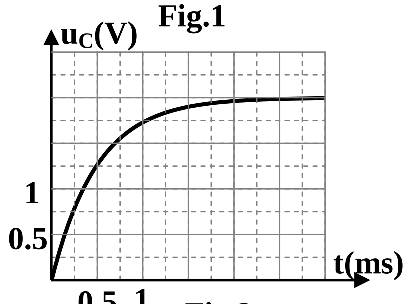


Fig.2

a) Determine the expression of τ in terms of R and C.
b) Show that at the end of duration 5τ, the charging of the capacitor is practically completely charged.

3) An appropriate system registers the variations of the voltage $u_C = u_{AB}$ across the terminals of the capacitor (Fig.2).
a) Referring to figure 2:
i. indicate the value of E;
ii. determine the value of τ;
b) Deduce the value of C.

B – RL circuit

The capacitor is replaced by a coil of inductance L and of resistance r (Fig.3).
At the instant $t_0 = 0$, the switch K is closed. An appropriate system records the variations of the current i in the circuit as a function of time (fig. 4).

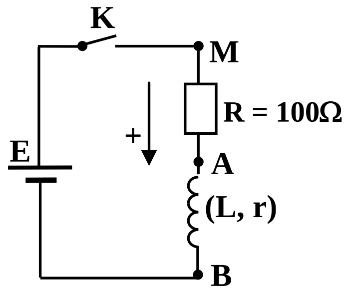


Fig.3

1) Derive the differential equation that describes the variation of i as a function of time.

2) Verify that: $i = \frac{E}{(R+r)}(1 - e^{\frac{-t}{\tau}})$ is the solution of the differential equation ,where $\tau = \frac{L}{R+r}$.

3) Determine, in the steady state, the expression of the current I in terms of E, R and r.

4) Referring to figure 4, indicate the value of I.

5) Determine the values of r and L.

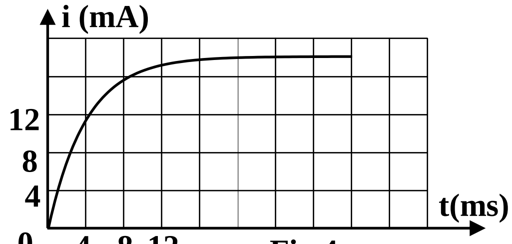


Fig.4

C – RLC circuit

The previous capacitor of capacitance $C = 5\times10^{-6}$ F, initially charged under the voltage E, is connected in series with the coil (L, r = 11Ω), the resistor of resistance R = 100Ω and the switch K as indicated in figure 5.
At $t_0 = 0$, the switch K is closed. The recording of the variations of the voltage $u_C = u_{AB}$ across the capacitor as a function of time is represented in figure 6.

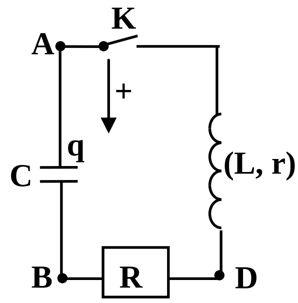


Fig.5

1) Derive the differential equation that describes the variation of u_C as a function of time.

2) The solution of this differential equation is:

$$u_C = 2\ e^{\frac{-(R+r)t}{2L}}\cos(\frac{2\pi}{T}t).$$

Use the graph of figure 6, to determine again the value of the inductance L found in part (B–5).

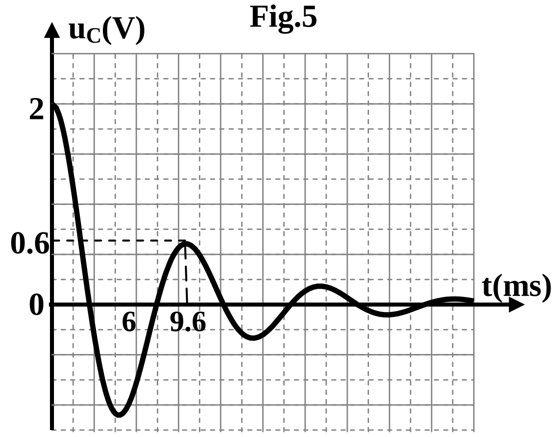


Fig.6

<u>Fourth exercise</u>: (7 ½ points) **Mechanical oscillations**

A simple pendulum consists of a particle of mass m = 100 g, fixed at the free end of a massless rod OA of length $\ell = 0.45$ m.
This pendulum may oscillate in the vertical plane, around a horizontal axis (Δ) passing through the upper extremity O of the rod.
The pendulum is initially at rest in its equilibrium position. At the instant $t_0 = 0$, the particle is launched horizontally in the positive direction as indicated in figure 1, with a velocity $\vec{v}_0$ of magnitude $v_0 = 0.3$ m/s.
At an instant t, the angular abscissa of the pendulum and the algebraic value of the velocity of the particle are θ and v respectively.

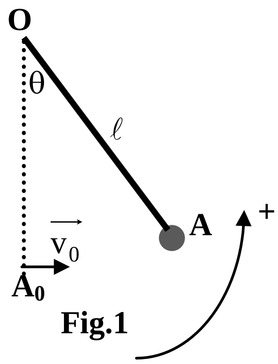


Fig.1

Take:

- The horizontal plane passing through A_0, the position of A at equilibrium, is taken as the reference level for the gravitational potential energy.
- $g = 10\ m/s^2$.
- For small angles: $\cos\theta = 1 - \frac{\theta^2}{2}$ and $\sin\theta \approx \theta$ where θ in rad.

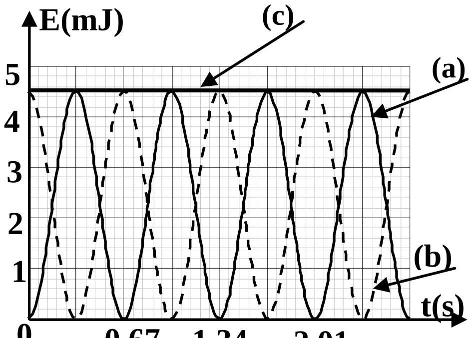


Fig.2

A – The forces of friction are negligible

1) **a)** Show that the mechanical energy of the system (pendulum, Earth) at the instant $t_0 = 0$ is $ME_0 = 4.5$ mJ.

b) Determine, at instant t, the expression of the mechanical energy of the system (pendulum, Earth) in terms of m, v, ℓ, g and θ.

c) Deduce the maximum angle θ_m performed by the pendulum.

2) **a)** Derive the differential equation in θ that describes the motion of the pendulum knowing that $v = \ell\frac{d\theta}{dt}$.

b) Deduce the expression of the proper angular frequency ω_0 and that of the proper period T_0 in terms of ℓ and g.

c) Calculate the values of T_0 and ω_0.

3) The time equation of the motion of the pendulum is of the form: $\theta = \theta_m \sin(\omega_0 t + \varphi)$. Determine φ.

4) Figure (2) shows three curves that represent the kinetic energy KE, the gravitational potential energy PE_g and the mechanical energy ME of the system (pendulum - Earth).

a) Identify each one of the curves a, b and c in the figure.

b) Pick up from figure 2 the value of the period T_E of the variations of the energies.

c) Deduce the relation between T_E and T_0.

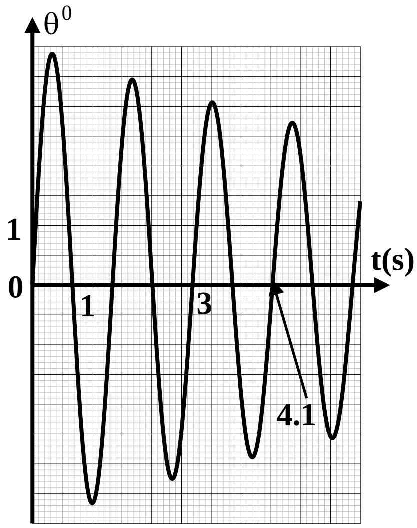


Fig.3

B – In reality, the forces of friction are not negligible. The variations of the angular abscissa θ of the pendulum as a function of time are represented by the graph of figure 3.

1) Referring to the graph:

a) indicate the type of oscillations performed by the pendulum;

b) determine the duration T of one oscillation. Compare T and T_0.

2) Knowing that the kinetic energy of the pendulum at the instant t = 2T is 2.74 mJ, determine the average power furnished to the pendulum in order to compensate the loss in energy between 0 and 2T.

C – The pendulum undergoes periodic excitations of adjustable angular frequency ω_e.

We record for each value of ω_e the value of the amplitude θ_m of the oscillations of the pendulum, and we trace the graph of $\theta_m = f(\omega_e)$ represented in figure 4.

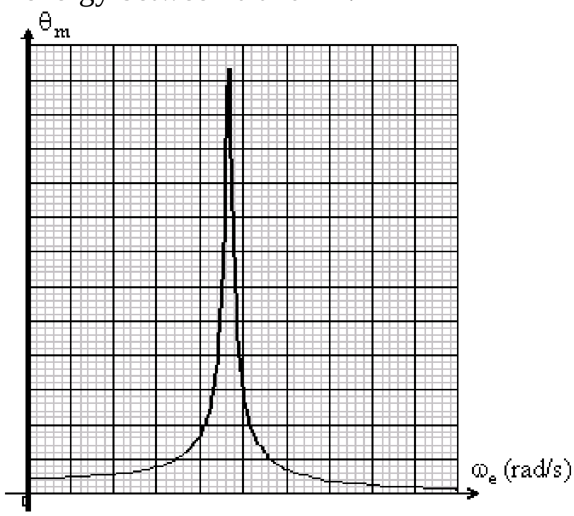
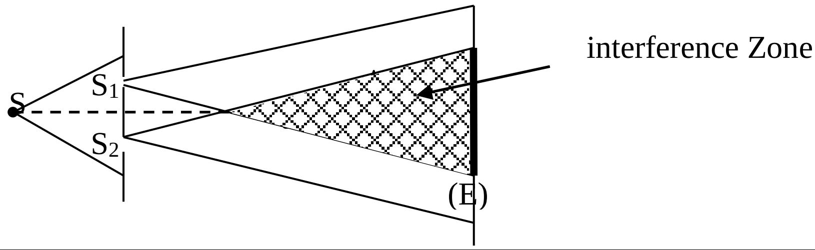


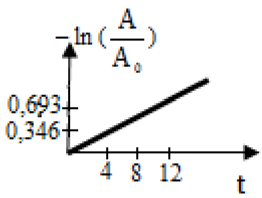
Fig.4

1) **a)** Name the phenomenon that takes place in the graph.

b) Give the value of the angular frequency ω_e so that the amplitude of oscillations is maximum.

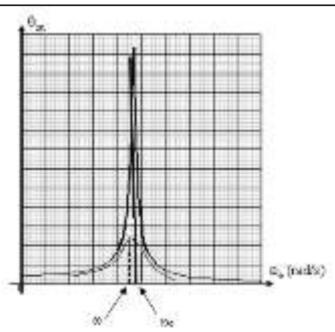
2) An appropriate system may increase slightly the forces of friction. Redraw figure 4 and draw roughly the shape of the curve giving the variations of the amplitude θ_m of oscillations of the pendulum in terms of the angular frequency ω_e of the excitations.

First exercise:Measurement of a small displacement [7.5 pts]		
A.1	$W_0 = 1.9\times1.6\times10^{-19} = \frac{hc}{\lambda_0} \Leftrightarrow \lambda_0 = 6.53 \times 10^{-7}$ m = 0.653 μm.	**0.75**
A.2.a	$KE_{max} = \frac{1}{2} m V^2_{max} = 2.56\times10^{-20}$ J	**0.5**
A.2.b	$\frac{hc}{\lambda_0} = W_0 + KE_{max} \Leftrightarrow \lambda_0 = 6.02\times10^{-7}$m = 0.602 μm	**0.75**
B.1		**0.5**
B.2.a	We observe on E alternating bright and darks fringes, rectilinear bands, parallel to the slits and equidistant centers.	**0.75**
B.2.b	$\delta = (SS_2 + S_2M) - (SS_1 + S_1M) = d_2 - d_1 = \frac{ax}{D}$	**0.25**
B.2.c.i	$\frac{ax}{D} = k\lambda$ (bright fringe) $\Leftrightarrow x_b = \frac{k\lambda D}{a}$	**0.5**
B.2.c.ii	$\frac{ax}{D} = (2k+1)\frac{\lambda}{2}$ (dark fringe) $\Leftrightarrow x_D = \frac{(2k+1)\lambda D}{2a}$	**0.5**
B.2.d	$\delta = \frac{ax}{D} = d_2 - d_1 = k\lambda$ corresponds to the bright fringes, at O, x = 0 then $\delta = 0 \Leftrightarrow k = 0$ O is the center of the central bright fringe is equidistant from S_1 and S_2.	**0. 5**
B.3.a	$i = x_{K+1} - x_K = \frac{\lambda D}{a}$; $i \approx 1.2\times10^{-3}$m	**0.75**
B.3.b	$x_D = \frac{(2k+1)\lambda D}{2a} \Leftrightarrow k = -4 \Leftrightarrow 4^{th}$ dark	**0.5**
B.4	When (S) is displaced along $\Delta \Leftrightarrow \delta_O = SS_2O - SS_1O = 0$ (unchanged) $\Leftrightarrow \delta_O = d_2 - d_1$ remain 0.	**0.5**
B.5	$\delta' = \frac{ay}{d} + \frac{ax}{D} = 0$ central fringe $\Leftrightarrow y = -\frac{xd}{D}$ the displacement of S is opposite of displacement of the central fringe. $y = -6 \times 10^{-6}$m ; S displaced by 6μm in the opposite direction of x	**0. 5**

<u>Second exercise</u> Nuclear power plant [7.5 pts]		
A.1.a	Fissile leads to fission	0.25
A.1.b	${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{94}_{Z}Sr + {}^{140}_{54}Xe + x\,{}^{1}_{0}n$ Apply the laws of conservation of mass number and charge number $\Rightarrow$ x =2 and Z=38	0.75
A.2	$E_{B\ (U)} = 7.5\times235 = 1762.5$ MeV $E_{B\ (Xe)} = 8.2\times140 = 1148$ MeV $E_{B\ (Sr)} = 8.5\times94 = 799$ MeV	0.25
A.3.a	$E_{B\ (X)} = [Z.m_p + (A-Z).m_n - m_X].\ C^2 \Rightarrow m_X = [Z.m_p + (A-Z).m_n] - E_B/c^2$	0.50
A.3.b	$E_{lib} = [(m_U + m_n) - (m_{Sr} + m_{Xe} + 2m_n)]c^2$; $m_X = Zm_P + (A-Z)m_n - E_B/c^2$ $E_{lib} = [(92m_P+143m_n-E_B(U)/c^2+m_n) - (38m_P+56m_n- E_B(Sr)/c^2+54m_P+86m_n-$ $E_B(Xe)/c^2+2m_n)]c^2$ $E_{lib} = E_B(Sr) + E_B(Xe) - E_l(U)$.	1.25
A.3.c	$E_{lib} = 184.5$ MeV	0.25
A.4	$P_e = 1350$ MW $\Leftrightarrow$ consumption of electric energy in one day : $E_e = P\times t = 1350\times10^{-6} \times 24\times3600 = 1.1664\times10^{14}$J total liberated energy due to fission = $E_t = \frac{E_e}{0.3} = 3.888\times10^{14}$ J = 2.43×10^{27} MeV 1 nucleus $\longrightarrow$ 200 MeV N $\longrightarrow$ 2.43×10^{27} MeV $N = 1.215\times10^{25}$ nuclei $m = \frac{N\times M}{N_A} = 4743$ g = 4.743 Kg	1.25
B.1.a	${}^{131}_{53}I \longrightarrow {}^{131}_{54}Te + {}^{0}_{-1}e + \bar{\nu} + \gamma$	0.5
B.1.b	The daughter nucleus (**Te**) is in an excited state	0.25
B.2.a	$\lambda = \ln2 / T = 0.693/8 = 0.0866$ day $^{-1}$	0.5
B.2.b	$A = A_0\, e^{-\lambda t} \Rightarrow A/A_0 = e^{-\lambda t} \Rightarrow -\ln(\frac{A}{A_0}) = \lambda t$	0. 5
B.2.c	Straight line passes through the origin 	0.75
B.2.d	$A = A_0/10 \Rightarrow t = 26.58$ days ≈ 27 days	0.5

Third exercise :Capacitor and coil [7.5 pts]		
Part of the Q	**Answer**	**Mark**
A.1	$E = Ri + u_C = RC\frac{du_C}{dt} + u_C$	**0.50**
A.2.a	$\frac{du_C}{dt} = \frac{E}{\tau}e^{\frac{-t}{\tau}} \Rightarrow E = RC\frac{E}{\tau}e^{\frac{-t}{\tau}} + E(1-e^{\frac{-t}{\tau}}) \Rightarrow \frac{RC}{\tau} = 1 \Rightarrow \tau = RC.$	**0.75**
A.2.b	If $t = 5\tau \Rightarrow u_C = E(1-e^{-5}) = 0.99E \approx E$	**0.50**
A.3.a.i	$E = 2$ V	**0.25**
A.3.a.ii	$t = \tau$, $u_C = 0.63\ E = 1.26\ V \Rightarrow \tau = 0.5$ ms.	**0.25**
A.3.b	$\tau = RC \Rightarrow C = 5\times10^{-6}$ F.	**0.50**
B.1	$E = Ri + ri + L\frac{di}{dt} \Rightarrow E = (R+r)i + L\frac{di}{dt}$.	**0.50**
B.2	$\frac{di}{dt} = \frac{E}{L}e^{\frac{-(R+r)t}{L}}$; $(R+r)\ \frac{E}{(R+r)}(1 - e^{\frac{-(R+r)t}{L}}) + L\frac{E}{L}e^{\frac{-(R+r)t}{L}} = E$ $\Rightarrow E = E$	**0.75**
B.3	If $t \to \infty \Rightarrow e^{\frac{-(R+r)t}{L}} \to 0 \Rightarrow I = \frac{E}{R+r}$ **<u>Or</u>** Steady state $i = I =$ cte $\Rightarrow L\frac{di}{dt} = 0 \Rightarrow E = (R+r)I + 0 \Rightarrow I = \frac{E}{R+r}$	**0.5**
B.4	$I = 18$ mA in steady state	**0.25**
B.5	$0.018 = \frac{2}{100+r} \Rightarrow r = 11\,\Omega$. At $t = \tau$; $i = 0.63\ I = 11.34$ mA $\Rightarrow \tau = 4$ ms. Or $\tau = \frac{L}{R+r} \Rightarrow L = 0.44$ H.	**1**
C.1	$u_{AB} + u_{BD} + u_{DA} = 0 \Rightarrow u_C + R.i + L\frac{di}{dt} + ri = 0$; $i = \frac{dq}{dt} = C\frac{du_C}{dt}$ $\Rightarrow u_C + (R+r).C\frac{du_C}{dt} + L.C\frac{d^2u_C}{dt^2} = 0.$	**0.75**
C.2	$t = 9.6$ ms , $u_C = 0.6$ V , we replace in the solution: $L = 0.44$ H	**0.75**

Fourth exercise :Mechanical oscillations [7.5 pts]

Part of the Q	Answer	Mark
A.1.a	$ME_0 = PE_g + KE = 0 + \frac{1}{2} m(v_0)^2 = 0 + \frac{1}{2}(0.1)(0.3)^2$ $ME_0 = 4.5\times10^{-3}$ J = 4.5 mJ.	**0.50**
A.1.b	$ME = \frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mg\ell(1-\cos\theta)$	**0.75**
A.1.c	ME (for $\theta = \theta_m$) = ME_0 (no friction) $\Rightarrow\ 0 + mg\ell(1-\cos\theta_m) = 0.1\times10\times0.45(1-\cos\theta_m) = 4.5\times10^{-3}$ J $\Rightarrow\ 1-\cos\theta_m = 0.01 \Rightarrow \cos\theta_m = 0.99 \Rightarrow \theta_m = 8^o$ $\Rightarrow$ the amplitude of oscillations being small	**0.75**
A.2.a	$\frac{dME}{dt} = 0 = mvv' + mg\,\theta'\,\ell\sin\theta$; we have $v = \ell\theta' \Rightarrow v' = \ell\,\theta''$, thus : $\theta'' + \frac{g}{\ell}\sin\theta = 0$, for small angles : $\sin\theta \approx \theta \Rightarrow \theta'' + \frac{g}{\ell}\theta = 0$	**0.75**
A.2.b	$\omega_0 = \sqrt{\frac{g}{\ell}}$ and $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$	**0.50**
A.2.c	$\omega_0 = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{10}{0.45}} = 4.71$ rd/s; $T_0 = 2\pi\sqrt{\frac{\ell}{g}} = 1.34$ s	**0.50**
A.3	$\theta = \theta_m\sin(\omega_0 t + \varphi)$; for t = 0, $\sin\varphi = 0 \Rightarrow \sin\varphi = 0 \Rightarrow \varphi = 0$ or $\varphi = \pi$ $\theta' = \omega_0\theta_m\cos(\omega_0 t + \varphi)$; $\theta'_0 = \omega_0\theta_m\cos\varphi > 0 \Rightarrow \varphi = 0$.	**0.75**
A.4	The curve (**c**) represents ME because it is parallel to t axis (not changed). Curve (**a**) represents PE_g because it passes in the origin at t = 0 (reference) The curve(**b**) represents KE because at $t = 0 \Rightarrow v_0 = v_{max} \Rightarrow KE_{max} = ME$	**0.5**
A.4.b	$T_E = 0.67$ s;	**0.25**
A.4.c	$T_0 = 1.34$ s = $2T_E$.	**0.25**
B.1.a	The pendulum performs free damped oscillations.	**0.25**
B.1.b	$T = \frac{4.1}{3} = 1.37$ s. T is slightly greater than T_0	**0.50**
B.2	$P_{av} = \frac{\lvert\Delta Em\rvert}{\Delta t} = \frac{\lvert ME_{(2T)} - ME_{(0)}\rvert}{2T}$ $ME_{(2T)} = KE_2 + PE_g = 2.74\times10^{-3} + 0 = 2.74\times10^{-3}$ J ; $ME_{(0)} = KE_0 + PE_g = 4.5\times10^{-3} + 0 = 4.5\times10^{-3}$ J ; To maintain the oscillations, it is necessary to provide to the pendulum an Average power: $P_{av} = \frac{1.76\times10^{-3}}{2\times1.37} = 0.64\times10^{-3}$ W	**0.75**
C.1.a	Amplitude Resonance	**0.25**
C.1.b	For $\omega_e = \omega_0 = 4.71$ rad/s	**0.25**
C.2.		**0.25**