This exam is formed of four exercises in four pages.
The use of non-programmable calculators is recommended.

First exercise: (7 points)

Mechanical oscillator

The aim of this exercise is to study the free oscillations of a mechanical oscillator. For this aim the oscillator is formed of a puck (A) of mass m = 0.25 kg fixed to one end of a massless spring of unjointed turns and of stiffness k = 10 N/m; the other end of the spring is attached to a fixed support (C) figure 1.

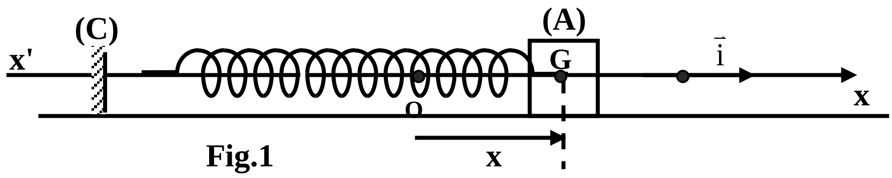


Fig.1

(A) slides on a horizontal rail and its center of inertia G can move on a horizontal axis x'ox.

At equilibrium, G coincides with the origin O of the axis x' x.

At an instant t the position of G is defined, on the axis (O, $\vec{i}$), by its abscissa $x = \overline{OG}$; its velocity $\vec{v} = v\,\vec{i}$ where $v = x' = \dfrac{dx}{dt}$.

The horizontal plane through G is taken as reference level for the gravitational potential energy.

A – Theoretical study

In this part we neglect all forces of friction.

1) **a)** Write down the expression of the mechanical energy of the system [(A) , spring , Earth] in terms of k , m , x and v .

b) Derive the differential equation in x that describes the motion of G.

2) The solution of this differential equation is of the form: $x = X_m \sin\left(\frac{2\pi}{T_0}t + \varphi\right)$ where X_m and φ are constants and T_0 is the proper period of the oscillator .

a) Determine the expression of T_0 in terms of m and k. Calculate its value.

b) At $t_0 = 0$, G passes through the point of abscissa $x_0 = 2$ cm with a velocity of algebraic value $v_0 = -0.2$ m/s, determine the values of X_m and φ .

B – Experimental study

In this part, the frictional force is given by $\vec{f} = -\mu\vec{v}$ where μ is a positive constant. An appropriate setup allow to record the curve giving the variations of x = f(t) (fig 2) and those giving the variations of the kinetic energy KE(t) of G and of the elastic potential energy PEe(t) of the spring (fig 3).

1) Referring to figure 2, give the value of the pseudo-period T of the motion of G .Compare its value to that of the proper period T_0.

2) Referring to figures 2 and 3, specify which curve A or B represents PEe(t).

3) **a)** Verify that the ratio $\dfrac{X_m(T)}{X_m(0)} = \dfrac{X_m(2T)}{X_m(T)} = a$ where a is constant to be determined.

b) Knowing that $a = e^{-\frac{\mu T}{2m}}$, calculate , in SI unit, the value of μ .

4) On figure 3 two particular instants t_1 and t_2 are located.

a) Referring to figure 3 indicate with justification at which instant , t_1 or t_2 , the magnitude of the velocity of the puck is :

i) maximum;

ii) equal to zero.

b) What can you conclude about the force of friction at each of the above instants?

c) Deduce around which instant t_1 or t_2, the mechanical energy decreases by a greater amount.

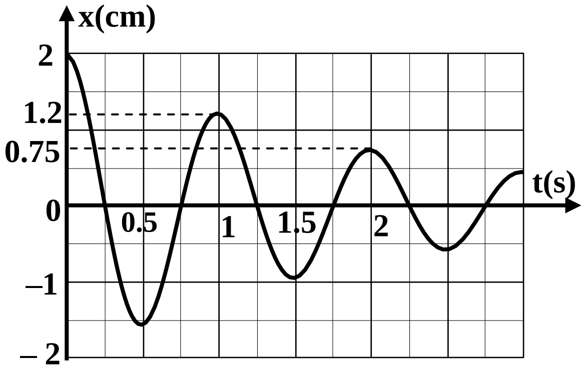


Fig.2

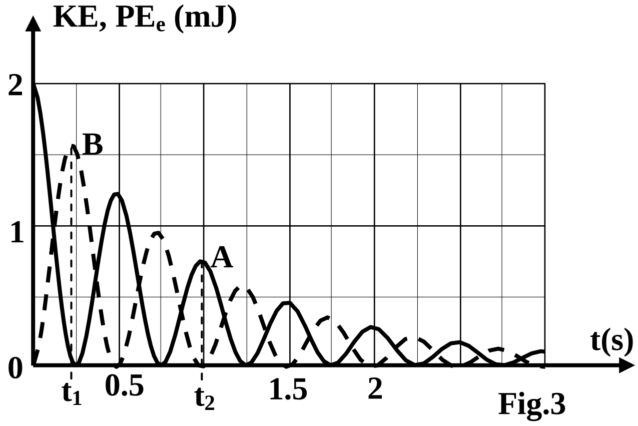


Fig.3

Second exercise: (7 points)

Characteristic of an electric component

In order to determine the characteristic of an electric component (D), we connect up the circuit represented in figure 1.

This series circuit is composed of: the component (D), a resistor of resistance $R = 100\,\Omega$, a coil (L = 25 mH; r = 0) and an (LFG) of adjustable frequency f maintaining across its terminals a sinusoidal alternating voltage $u = u_{AM}$.

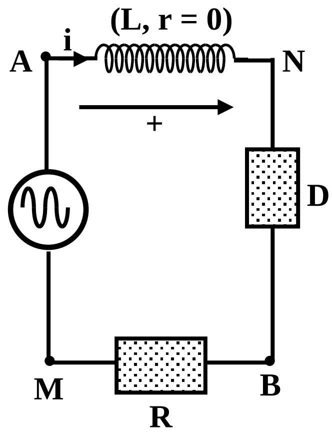


Fig.1

A – First experiment

We connect an oscilloscope so as to display the variation, as a function of time, the voltage u_{AM} across the generator on the channel (Y_1) and the voltage u_{BM} across the resistor on the channel (Y_2).

For a certain value of f , we observe the waveforms of figure 2.

The adjustments of the oscilloscope are:

- ✓ vertical sensitivity: 2 V/div on the channel (Y_1);
  0.5 V/div on the channel (Y_2);
- ✓ horizontal sensitivity: 1 ms/ div.

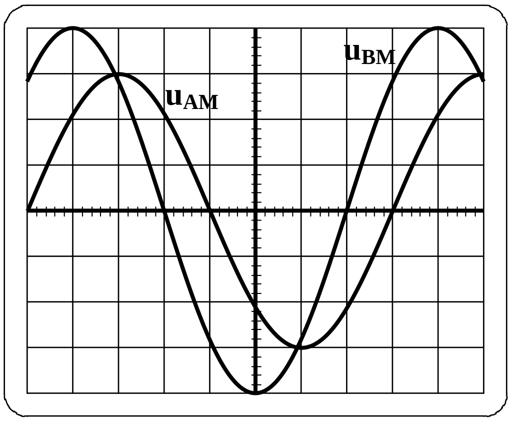


Fig.2

1) Redraw figure 1 and show on it the connections of the oscilloscope.

2) Using figure 2, determine:

a) the value of f and deduce the value of the angular frequency ω of u_{AM};

b) the maximum value U_m of the voltage u_{AM};

c) the maximum value I_m of the current i in the circuit;

d) the phase difference φ between u_{AM} and i. Indicate which one leads the other.

3) (D) is a capacitor of capacitance C. Justify.

4) Given that: $u_{AM} = U_m \sin \omega t$. Write down the expression of i as a function of time.

5) Show that the expression of the voltage across the capacitor is:

$$u_{NB} = -\frac{0.02}{250\pi C}\cos(\omega t + \frac{\pi}{4}) \quad (u_{NB} \text{ in V ; C in F ; t in s})$$

6) Applying the law of addition of voltages and by giving t a particular value, determine the value of C.

B – Second experiment

The effective voltage across the generator is kept constant and we vary the frequency f . We record for each value of f the value of the effective current I.

For a particular value $f = f_0 = \frac{1000}{\pi}$ Hz , we notice that I admits a maximum value.

1) Name the phenomenon that takes place in the circuit for the frequency $f = f_0$.

2) Determine again the value of C.

<u>Third exercise</u>: (6 ½ points)

Electric circuit

A – Study of a (L, C) circuit

The (L,C) circuit of figure 1, composed of a capacitor of capacitance C and of a coil of inductance L and of negligible resistance and a switch k. Initially the armature A of C caries a charge $Q_0 > 0$. The switch is closed at $t_0 = 0$. At an instant t the charge of armature A is q and the current in the circuit is i.

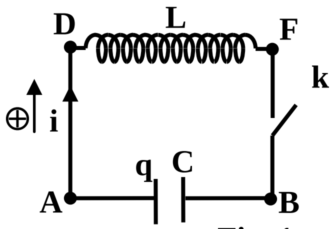


Fig. 1

1) **a)** Indicate the form of the energy in the circuit at $t_0 = 0$.

b) Deduce that the value of the current at $t_0 = 0$ is 0.

2) Using the principle of the conservation of the electromagnetic energy, show that the differential equation in q has the form of: $q'' + \frac{1}{LC}q = 0$.

3) The solution of this differential equation is of the form $q = Q_m \cos(\omega_0 t + \phi)$; Q_m, ω_0 and ϕ are constants and $Q_m > 0$.

a) Determine ϕ.

b) Determine the expression of Q_m in terms of of Q_0 and that of ω_0 in terms of L and C.

4) **a)** Determine the expression of i as a function of time.

b) Trace the shape of the curve i = f(t).

B – Study of a (L, R) series circuit

We consider the adjacent circuit 2. It is formed of an ideal DC power supply of e.m.f. E, a resistor of resistance R, a switch k and a coil of inductance L and of zero internal resistance.

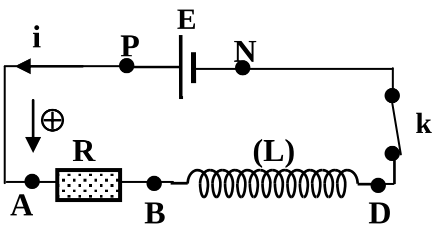


Fig.2

The switch is closed at $t_o = 0$.

1) Apply the law of addition of voltages, establish the differential equation in the current i.

2) Deduce that the value of the current in the steady state is $I = \frac{E}{R}$.

3) The solution of the differential equation is of the form: $i = A + B e^{-\lambda t}$. Determine the expression of the constants A, B and λ in terms of I , R and L.

4) When k is opened we observe a spark on k.

a) Name the phenomenon responsible of this spark.

b) To eliminate this spark a neutral capacitor is used. How should it be connected?

<u>Fourth exercise</u>: (7 points)

Nuclear reactions

Given: mass of a proton: m_p = 1.0073 u; mass of a neutron: m_n = 1.0087 u;
mass of $^{235}_{92}U$ nucleus = 235.0439 u; mass of $^{90}_{36}Kr$ nucleus = 89.9197 u;
mass of $^{142}_{Z}Ba$ nucleus =141.9164 u; molar mass of $^{235}_{92}U$ = 235 g/mole;
Avogadro's number: $N_A = 6.022\times10^{23}$ mol^{-1}; 1u = 931.5 MeV/c^2 = 1.66×10^{-27} kg; 1 MeV = 1.6×10^{-13} J.

A – Provoked nuclear reaction

As a result of collision with a thermal neutron, a uranium 235 nucleus undergoes the following reaction:

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{90}_{36}Kr + ^{142}_{Z}Ba + y\,^{1}_{0}n$$

1) **a)** Determine y and z.
b) Indicate the type of this provoked nuclear reaction.

2) Calculate, in MeV, the energy liberated by this reaction.

3) In fact, 7% of this energy appears as a kinetic energy of all the produced neutrons.
a) Determine the speed of each neutron knowing that they have equal kinetic energy.
b) A thermal neutron, that can provoke nuclear fission, must have a speed of few km/s; indicate then the role of the “moderator” in a nuclear reactor.

4) In a nuclear reactor with uranium 235, the average energy liberated by the fission of one nucleus is 170 MeV.
a) Determine, in joules, the average energy liberated by the fission of one kg of uranium $^{235}_{92}U$.
b) The nuclear power of such reactor is 100 MW. Calculate the time Δt needed so that the reactor consumes one kg of uranium $^{235}_{92}U$.

B – Spontaneous nuclear reaction

1) The nucleus krypton $^{90}_{36}Kr$ obtained is radioactive. It disintegrates into zirconium $^{90}_{40}Zr$, by a series of β^- disintegrations.
a) Determine the number of β^- disintegrations.
b) Specify, without calculation, which one of the two nuclides $^{90}_{36}Kr$ and $^{90}_{40}Zr$ is more stable.

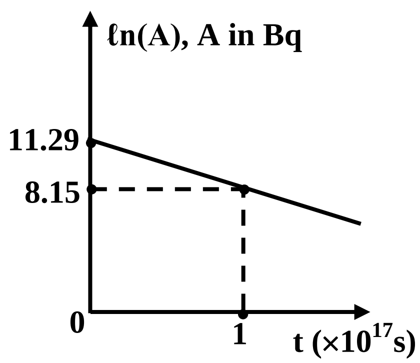
2) Uranium $^{235}_{92}U$ is an α emitter.
a) Write down the equation of disintegration of uranium $^{235}_{92}U$ and identify the nucleus produced.
Given:

Actinium $_{89}Ac$	Thorium $_{90}Th$	Protactinium $_{91}Pa$

b) The remaining number of nuclei of $^{235}_{92}U$ as a function of time is given by: $N = N_0 e^{-\lambda t}$ where N_0 is the number of the nuclei of $^{235}_{92}U$ at $t_0 = 0$ and λ is the decay constant of $^{235}_{92}U$.
i) Define the activity A of a radioactive sample.
ii) Write the expression of A in terms of λ, N_0 and time t.

c) Derive the expression of $\ell n(A)$ in terms of the initial activity A_0, λ and t.

d) The adjacent figure represents the variation of $\ell n(A)$ of a sample of $^{235}_{92}U$ as a function of time.



i) Show that the shape of the graph, in the adjacent figure, agrees with the expression of $\ell n(A)$.
ii) Using the adjacent figure determine, in s^{-1}, the value of the radioactive constant λ.
iii) Deduce the value of the radioactive period T of $^{235}_{92}U$.

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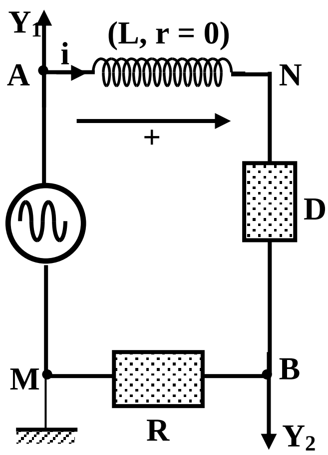
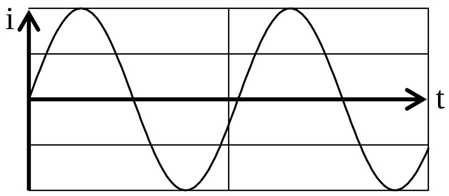
First exercise (7 points)

Part of the Q	Answer	Mark
A.1.a	ME = $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$	½
A.1.b	No frictional forces (No non-conservative forces), ME is conserved $\Rightarrow \frac{dME}{dt} = 0 \Rightarrow x'' + \frac{k}{m}x = 0$.	¾
A.2.a	$x = X_m \sin(\frac{2\pi}{T_o}t + \varphi)$ $x' = X_m \frac{2\pi}{T_o}\cos(\frac{2\pi}{T_o}t + \varphi)$; $x'' = -X_m \frac{4\pi^2}{T_0^2}\sin(\frac{2\pi}{T_o}t + \varphi)$ Sub. in the differential equation : $-X_m \frac{4\pi^2}{T_0^2}\sin(\frac{2\pi}{T_o}t + \varphi) + \frac{k}{m}X_m \sin(\frac{2\pi}{T_o}t + \varphi) = 0$ $\Rightarrow \frac{4\pi^2}{T_0^2} = \frac{k}{m} \Rightarrow T_o = 2\pi\sqrt{\frac{m}{k}} = 2\times 3.14\sqrt{\frac{0.25}{10}} = 0.993\text{ s} \approx 1\text{ s}$	1^+
A.2.b	For t = 0 : x = 2 cm $\Rightarrow 2 = X_m \sin\varphi$ v = – 20 cm/s $\Rightarrow -20 = X_m \times 2\pi\cos\varphi$ The ratio gives : $\tan\varphi = -0.628 \Rightarrow \varphi = -0.56$rd or 2.58 rad For $\varphi = -0.56$rd we get : $X_m = -3.77$ cm < 0 rejected For $\varphi = 2.58$rd we get : $X_m = 3.77$ cm > accepted Therefore $X_m = 3.77$ cm and $\varphi = 2.58$ rad .	1^+
B.1	The graph gives T = 1 s slightly greater than $T_o = 0.993$ s.	½
B.2	On the graph of figure 2 , at $t_0 = 0$, x is maximum therefore PEe ≠ 0 $\Rightarrow$ curve A represents the variations of PEe.	½
B.3.a	$\frac{X_m(T)}{X_m(0)} = \frac{12.5}{20} = 0.625$; $\frac{X_m(2T)}{X_m(T)} = \frac{7.5}{12.5} = 0.6 \Rightarrow a = 0.6$	½
B.3.b	$a = e^{-\frac{\mu T}{2m}} = 0.6 \Rightarrow \frac{-\mu T}{2m} = \ell n0.6 \Rightarrow \mu = 0.255$ kg/s.	¾
B.4.a.i	At instant t_1, the KE is maximum therefore v has a maximum value.	+
B.4.a.ii	At instant t_2 , the KE is equal to zero $\Rightarrow$ v has a zero value.	+
B.4.b	At instant t_1, f has large amplitude (v maximum). At instant t_2, f is equal to zero (v = 0) .	½
B.4.c	Around (t_1) the force of friction is maximum $\Rightarrow$ the mechanical energy decreases by greater value.	½

Second exercise (7 points)

Part of the Q	Answer	Mark
A.1		½
A.2.a	T = 8 ms $\Rightarrow$ f = 125 Hz. $\omega = 2\pi f = 250\pi$ rad/s.	**1**
A.2.b	$U_m = 3 \times 2 = 6$ V.	+
A.2.c	$U_{m(R)} = 0.5 \times 4 = 2$ V $\Rightarrow I_m = \frac{U_m(R)}{R} = 2\times 10^{-2}$ A	¾
A.2.d	$\lvert\varphi\rvert = \frac{2\pi}{8} = \frac{\pi}{4}$ rad ; i leads u_{AM}	¾
A.3	i leads u_{AM} $\Rightarrow$ (D) is a capcitor	+
A.4	$i = 2\times 10^{-2} \sin(250\pi t + \frac{\pi}{4})$ (i in A and t in s)	½
A.5	$i = C\frac{du_{BN}}{dt} \Rightarrow u_{NB} = \frac{1}{C}\int i\,dt = \frac{1}{C}\int 0.02\sin(\omega t + \frac{\pi}{4})dt$ $\Rightarrow u_{NB} = -\frac{0.02}{250\pi C}\cos(250\pi t + \frac{\pi}{4})$	¾
A.6	$U_m\sin(\omega t) = L\omega I_m \cos(\omega t + \frac{\pi}{4}) - \frac{0.02}{250\pi C}\cos(250\pi t + \frac{\pi}{4}) + 2\sin(\omega t + \frac{\pi}{4})$ $t = 0 \Rightarrow 0 = L\omega I_m \frac{\sqrt{2}}{2} - \frac{0.02}{250\pi C}\times\frac{\sqrt{2}}{2} + 2\frac{\sqrt{2}}{2} \Rightarrow C = 1.06\times 10^{-6}$ F	**1.25**
B.1	Current resonance	+
B.2	$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = 1.06\times 10^{-6}$ F	¾

Third exercise (6 ½ points)

Partie de la Q.	Corrigé	Note
A.1.a	Electric energy	+
A.1.b	$E_{mag} = 0 = ½ Li^2 \Rightarrow i = 0$	½
A.2	At instant t : $E_{total} = E_{elect} + E_{mag} = ½ q^2/C + ½ Li^2 = constant$ With $i = -\frac{dq}{dt} = -q'$ and $i' = -\frac{d^2q}{dt^2} = -q''$ $\frac{dE}{dt} = 0 \Rightarrow \frac{qq'}{C} + Lii' = 0$ $\Rightarrow \frac{qq'}{C} + L(-q')(-q'') = 0 \Rightarrow q'(\frac{q}{C} + Lq'') = 0$ or $q' \neq 0 \Rightarrow q'' + \frac{1}{LC}q = 0$	¾
A.3.a	$q = Q_m \cos(\omega_0 t + \phi) \Rightarrow \dot{q} = -\omega_0 Q_m \sin(\omega_0 t + \phi)$ $\Rightarrow \ddot{q} = -Q_m(\omega_0)^2 \cos(\omega_0 t + \phi)$; $i = -\frac{dq}{dt} = \omega_0 Q_m \sin(\omega_0 t + \phi)$; for t = 0 , i = 0 $\Rightarrow 0 = \sin\phi = 0$; $\Rightarrow \phi = 0$ or π rad; for t= 0 $q = Q_0 > 0 = Q_m \cos\phi$, with $Q_m > 0 \Rightarrow \cos\phi > 0 \Rightarrow \phi = 0$.	**1**
A.3.b	$Q_0 = Q_m \cos\phi \Rightarrow Q_m = Q_0$. Replace $q = Q_0 \cos\omega_0 t$ in the differential equation: $-Q_0(\omega_0)^2 \cos(\omega_0 t) + Q_0 \frac{1}{LC}\cos(\omega_0 t) = 0 \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \sqrt{\frac{1}{LC}}$	¾
A.4.a	$i = -\frac{dq}{dt} = \omega_0 Q_m \sin(\omega_0 t + \phi) \Rightarrow i = \omega_0 Q_m \sin(\omega_0 t)$	¾
A.4.b		¼
B.1	$u_{AD} = u_{AB} + u_{BD} \Rightarrow E = Ri + L\frac{di}{dt}$,	½
B.2	$E = Ri + L\frac{di}{dt}$, with $u_C = L\frac{di}{dt}$; at steady state, i = constant = I $\Rightarrow \frac{di}{dt} = 0 \Rightarrow E = RI$ and $I = \frac{E}{R}$.	½
B.3	$i = A + B.e^{-\lambda t}$ At $t_0 = 0$: $A + B = i = 0 \Rightarrow A = -B$ $\frac{di}{dt} = -\lambda B e^{-\lambda t} \Rightarrow E = RA - RAe^{-\lambda t} + L\lambda A e^{-\lambda t} \Rightarrow Ae^{-\lambda t}(L\lambda - R) + RA = E$ by identification : $L\lambda - R = 0 \Rightarrow \lambda = \frac{R}{L}$ and $RA = E \Rightarrow A = \frac{E}{R}$ $\Rightarrow B = -A = -\frac{E}{R}$	**1**
B.4.a	Self - induction	+
B.4.b	Across the switch k	+

Fourth exercise (7 points)

Part of the Q	Answer	Mark
A.1.a	Conservation of charge number: 92 + 0 = 36 + z + 0 thus z = 56 Conservation of mass number: 235 + 1 = 90 + 142 + y thus y = 4	¾
A.1.b	Fission nuclear reaction	¼
A.2	$\Delta m = [m_U + m_n] - [m_{Kr} + m_{Ba} + 4m_n]$ $= 235.0439 - [89.9197 + 141.9164 + 3\times1.0087] = 0.1817$ u $E = \Delta mc^2 = [0.1817\times931.5\ \text{Mev}/c^2]\ c^2 = 169.253$ MeV	¾
A.3.a	K.E of each neutron = $\frac{169.253\times\frac{7}{100}}{4}$ = 2.96 MeV = $2.96\times1.6\times10^{-13}$ K.E = 4.739×10^{-13} J K.E = ½ mV^2 then V = $\sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\times4.739\times10^{-13}}{1.0087\times1.66\times10^{-27}}}$ V = 2.379×10^{7} m/s = 23790 km/s.	½
A.3.b	A moderator will help in reducing their speed so as to provoke more such reactions	¼
A.4.a	N = $\frac{\text{mass}}{\text{molar mass}}\times N_A = \frac{1000}{235}\times6.02\times10^{23} = 2.5617\times10^{24}$ nuclei. E = $170\times1.6\times10^{-13}\times2.5617\times10^{24} = 6.97\times10^{13}$ J	½
A.4.b	$E = P\times\Delta t \Rightarrow \Delta t = \frac{6.97\times10^{13}}{10^{8}} = 6.97\times10^{5}$ s = 8 days	½
B.1.a	${}^{90}_{36}Kr \rightarrow {}^{90}_{40}Zr + a\,{}^{0}_{-1}\beta$ a = 4	¼
B.1.b	A non-stable nucleus decays into a more stable one thus ${}^{90}_{40}Zr$ is more stable	¼
B.2.a	${}^{235}_{92}U \rightarrow {}^{4}_{2}He + {}^{A}_{Z}X$, A = 231 and Z = 90 ⇒X is thorium	½
B.2.b.i	The activity is the number of decays per unit time	¼
B.2.b.ii	$A = \lambda N = \lambda N_0 e^{-\lambda t}$	¼
B.2.c	$\ell n(A) = -\lambda t + \ell n(A_0)$.	½
B.2.d.i	$\ell n(A) = -\lambda t + \ell n(A_0)$ is a straight line of negative slope ⇒ compatible with the graph.	½
B.2.d.ii	λ = – slope of curve = $3.14\times10^{-17}\ s^{-1}$,	½
B.2.d.iii	$\lambda = \frac{\ell n(2)}{T} \Rightarrow T = 22.0747\times10^{15}$s = 7×10^{8} years.	½