امتحانات الشهادة الثانوية العامة
الفرع: علوم عامة

دورة الـعام 2017 العاديّة
السبت 17 حزيران 2017

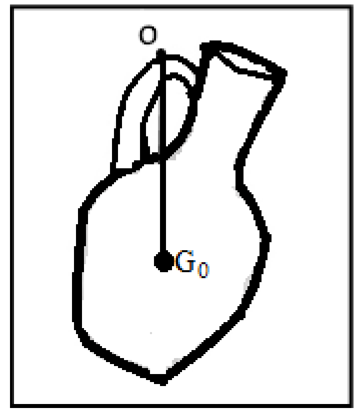
مسابقة في مادة الفيزياء
المدة: ثلاث ساعات

الاسم:
الرقم:

This exam is formed of four exercises in four pages.
The use of non-programmable calculator is recommended.

Exercise 1 (8 points) Determination of the moment of inertia of a pottery vase

The aim of this exercise is to determine the moment of inertia of a pottery vase about two different axis of rotation. The vase has a mass m = 2 kg and center of mass G.



Doc. 1

1- Moment of inertia of the vase about a horizontal axis

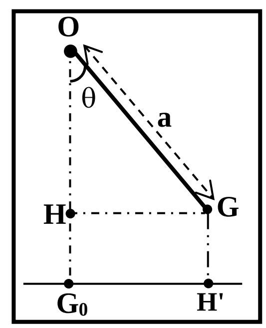
We suspend the vase from a point O, such that the vase is a compound pendulum which can oscillate freely, without friction, about a horizontal axis (Δ) passing through O (Doc 1).

The moment of inertia of the vase about (Δ) is I.

At equilibrium, the center of mass of the vase is in the position G_0, directly below the suspension point O ($OG = OG_0 = a = 24$ cm).

The vase is displaced from its stable equilibrium position by a small angle $\theta_m = 0.16$ rad, and then it is released from rest.

Document 2 is a simplified diagram of the compound pendulum at an instant t.



Doc. 2

At the instant t, the angular abscissa of G is $\theta = (\overrightarrow{OG_0}, \overrightarrow{OG})$ and the algebraic value of its angular velocity is $\theta' = \frac{d\theta}{dt}$.

The horizontal plane passing through G_0 is taken as a gravitational potential energy reference.

Neglect air resistance.

Given: $g = 10$ m/s^2 ; for small angles: $\cos\theta = 1 - \frac{\theta^2}{2}$ and $\sin\theta = \theta$ (θ in rad).

1-1) Determine, at an instant t, the expression of the mechanical energy of the system (pendulum – Earth) in terms of I, a, g, m, θ and θ'.

1-2) Establish the differential equation in θ that describes the motion of the pendulum.

1-3) The solution of the obtained differential equation is: $\theta = \theta_m \sin(\omega_0 t + \varphi)$. θ_m, φ and ω_0 are constants.

1-3-1) Determine the expression of the proper angular frequency ω_0.

1-3-2) Deduce the expression of the proper period T_0 of the oscillations of the pendulum in terms of I, m, g and a.

1-4) The pendulum completes 9 oscillations in 25.2 seconds.

1-4-1) Calculate the proper period T_0 of the oscillations.

1-4-2) Deduce the value of I.

1-5) An appropriate device measures the angular speed of the pendulum. The angular speed of the pendulum when it passes in its equilibrium position is $\theta'_{eq} = 0.36$ rad/s. Apply the principle of conservation of mechanical energy for the system (pendulum, Earth) to determine again the value of I.

2- Moment of inertia of the vase about a vertical axis

Consider a horizontal turntable rotating clockwise at an angular speed of $\theta'_t = 0.7$ rad/s about a vertical axis (Δ') passing through its center of mass.
The mass of the table is M = 20 kg and its radius is R = 50 cm.
Slowly, we put the vase on the rim of the turntable.
The system (turntable - vase) rotates clockwise with an angular speed of $\theta'_{system} = 0.45$ rad/s.

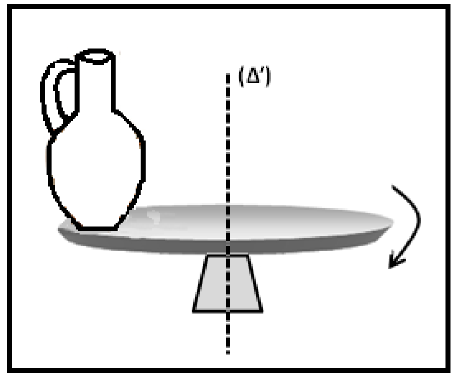
The moment of inertia of the table about (Δ') is: $I_t = \frac{1}{2}MR^2$.

The moment of inertia of the vase about (Δ') is I'.

2-1) Name the external forces acting on the system (turntable-vase).

2-2) Show that the angular momentum σ, about (Δ'), of the system (turntable- vase) is conserved.

2-3) Deduce the value of I'.



Doc. 3

<u>Exercise 2</u> (7 ½ points) **Sodium atom**

Document 1 represents some of the energy levels of the sodium atom.

Given: $h = 6.6 \times 10^{-34}$ J.s ; $c = 3 \times 10^{8}$ m/s ;
$1 \text{ eV} = 1.6 \times 10^{-19}$ J ; $1 \text{ u} = 931.5$ MeV/c^2.

The aim of this exercise is to study the excitation and the de-excitation of the sodium atom.

1- Excitation of the sodium atom

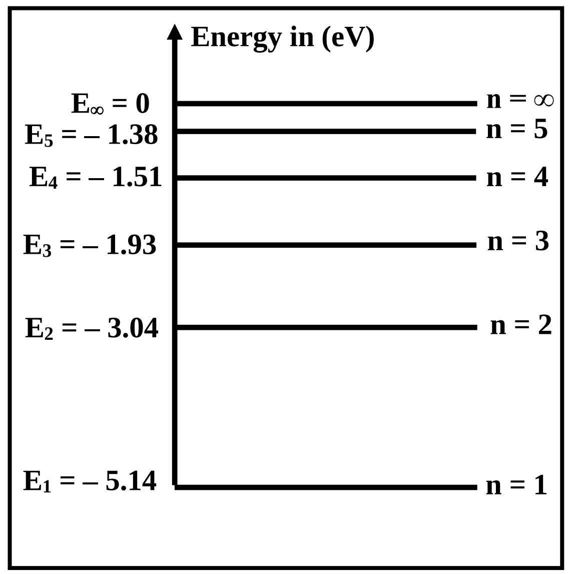
Consider a sample of sodium atoms, initially in the ground state. This sample is illuminated by white light that contains all the visible radiations: $0.4\ \mu m \leq \lambda_{visible} \leq 0.8\ \mu m$.

1-1) Using document 1, show that the energy of the sodium atom is quantized.

1-2) Determine, in eV, the maximum energy and the minimum energy of the photons in the white light.

1-3) Using document 1, show that white light is not capable to ionize the sodium atom.

1-4) Determine, in nm, the wavelength of the photon that excites the sodium atom to the first excited state.



Doc. 1

2- De-excitation of the sodium atom

The emission spectrum, obtained from the low-pressure sodium vapor lamp, contains two very close yellow lines of wavelengths $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm, called the D-doublet of sodium.

2-1) The sodium atom de-excites from the energy level E_n to the ground state and emits the photon of wavelength $\lambda_1 = 589.0$ nm. Specify the value of E_n in eV.

2-2) The sodium atom undergoes a transition from the energy level E_3 to the energy level E_1.
During this transition it loses energy $E_{3\to1}$ and its mass decreases by Δm.

2-2-1) Calculate, in MeV, the value of $E_{3\to1}$.

2-2-2) Deduce, in u, the value of Δm.

2-3) The power of the radiations of wavelengths λ_1 and λ_2 emitted by the sodium vapor lamp is P = 6 W.
The power P_1 of the radiation of wavelengths λ_1 is twice the power P_2 of the radiation of wavelengths λ_2.

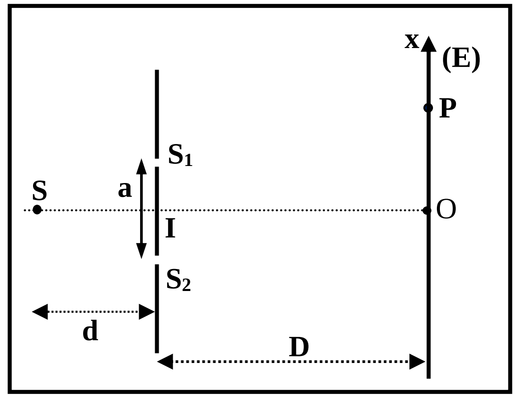
2-3-1) Show that $P_1 = 4$ W.

2-3-2) Determine the number of photons of the radiation of wavelength λ_1 emitted from the sodium vapor lamp in one second.

<u>Exercise 3</u> (7 points) **Interference of light**

Document1 shows the set-up of Young's double- slit experiment.
(OI) is the perpendicular bisector to $[S_1S_2]$.
A point source S, emitting a monochromatic light of wavelength $\lambda = 500$ nm in air, is placed in front of the two slits S_1 and S_2.
P is a point on the interference pattern on a screen (E), and it has an abscissa $x = \overline{OP}$ relative to the origin O of the x-axis. The distance between S_1 and S_2 is "a", and the distance between the plane of the slits and the screen (E) is D.

Given: $S_2P - S_1P = \frac{a\,x}{D}$.

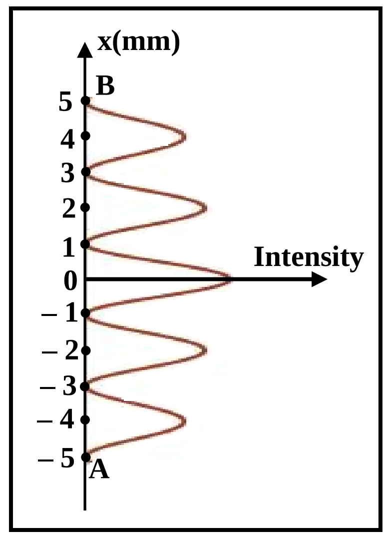


Doc. 1

The optical path difference at the point P is $\delta = SS_2P - SS_1P$.
The aim of this exercise is to determine "a" and D.

1- S is placed on the line (IO) as shown in document 1. In this case the optical path difference at the point P is $\delta = \frac{a\,x}{D}$.

1-1) Show that the point O is the center of the central bright fringe.
1-2) Determine the expression of the abscissa of the center of the k^{th} dark fringe.
1-3) Deduce the expression of the inter-fringe distance i, in terms of a, λ and D.
1-4) An appropriate device records the intensity of the light received from S on the screen (E) as a function of x. The graph of document 2 shows the intensity as a function of x between two points A and B.
Refer to document 2:
1-4-1) indicate the number of bright fringes between A and B;
1-4-2) give the expression of the distance AB in terms of the inter-fringe distance i;
1-4-3) indicate the order and nature of the fringe whose center is the point B;
1-4-4) give the abscissa of the center of the first dark fringe on the positive side of O.



Doc. 2

1-5) Deduce that D = 4000 a (in SI units).

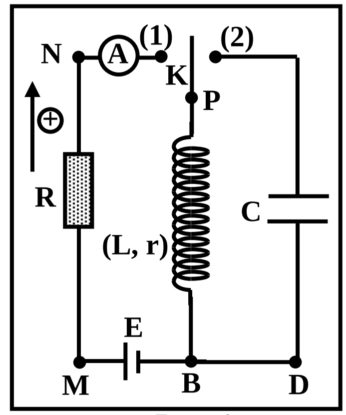
2- The point source S which is placed at a distance "d" from the plane of the slits is moved by a displacement z to the side of S_1 in a direction perpendicular to (IO) and normal to the slits.

Given: $SS_2 - SS_1 = \frac{a\,z}{d}$.

2-1) Prove that the optical path difference of the point P is $\delta = \frac{a\,z}{d} + \frac{a\,x}{D}$.
2-2) Deduce the expression of the abscissa of the center of the central bright fringe.
2-3) We notice that the center of the central bright fringe coincides with the position that was occupied by the center of the 10^{th} bright fringe, on the negative side of O, before the displacement of S.
Given: d = 40 cm and z = 0.4 cm.
Determine the values of a and D.

Exercise 4 **(7 ½ points)** **Characteristics of a coil**

The aim of this exercise is to determine the characteristics of a coil. For this aim, consider the circuit of document 1 which includes a coil of inductance L and resistance r, an initially neutral capacitor of capacitance C, an ideal DC generator of e.m.f E, a resistor of resistance R, a double switch K, and an ammeter (A) of negligible resistance.



Doc. 1

1- First experiment

K is put at position (1) at $t_0 = 0$. The ammeter (A) indicates a current i which increases from zero to its maximum value $I_0 = 0.1$ A and the steady state is attained.

1-1) Name the phenomenon that takes place in the coil during the growth of the current.

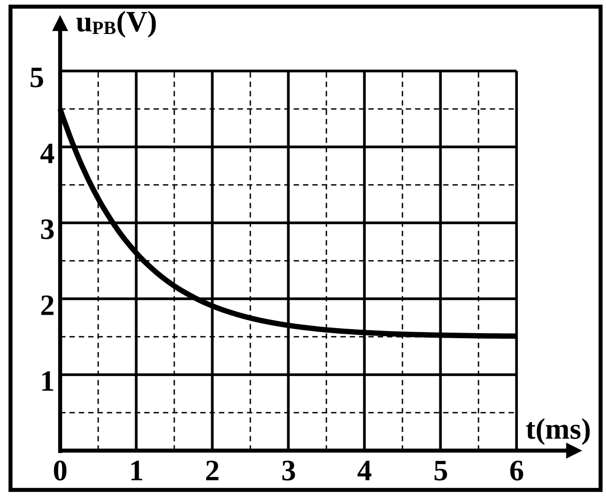
1-2) Determine, using the law of addition of voltages, the expression of I_0 in terms of E, R and r.

1-3) A suitable device allows us to record the voltage u_{PB} between the terminals of the coil as a function of time as indicated by the curve of document 2.

1-3-1) Applying the law of addition of voltages, and using the curve of document 2 , show that E = 4.5 V.

1-3-2) Using document 2, prove, without calculation that the value of r is not zero.

1-3-3) Deduce that r = 15 Ω.



Doc.2

1-4) Show that R = 30 Ω.

1-5) Establish, by applying the law of addition of voltages, the differential equation that describes the variation of the current i as a function of time.

1-6) The solution of this differential equation has the form:

$i = I_0(1 - e^{\frac{-t}{\tau}})$, where τ is constant.

1-6-1) Determine the expression of τ in terms of L, r and R.

1-6-2) Determine at t = τ the value of the voltage $u_R = u_{MN}$ across the resistor.

1-6-3) Show, at t = τ, that the voltage across the coil is $u_{PB} = 2.61$ V.

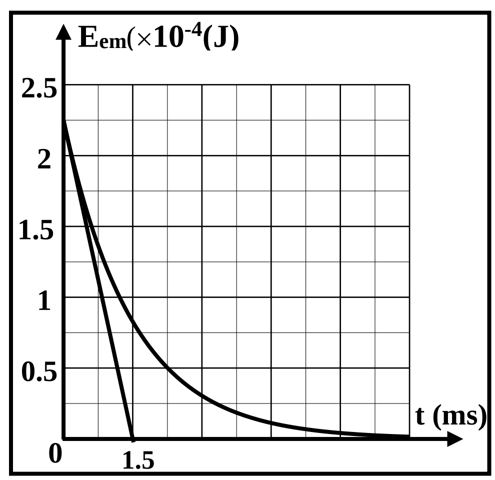
1-6-4) Deduce, using document 2, the value of τ.

1-7) Calculate the value of L.

2- Second experiment

When the steady state of the current in the coil is attained ($i = I_0$), K is moved abruptly from position (1) to position (2) at an instant $t_0 = 0$ taken as a new origin of time. The electromagnetic energy in the circuit at an instant t is $E_{em} = E_{electric} + E_{magnetic}$.

An appropriate device allows us, to trace the curve of the electromagnetic energy as a function of time and the tangent to this curve at $t_0 = 0$ (Doc. 3).



Doc. 3

2-1) Using document 3, indicate the value of E_{em} at $t_0 = 0$.

2-2) Deduce the value of L.

2-3) Calculate the slope of the above tangent.

2-4) Deduce the value of r, knowing that $\frac{dE_{em}}{dt} = - r\, i^2$.

<u>Exercise 1</u> (8 points)

Part			Answer	Mark
1	1-1		GPE = m g h_G . But h_G = GH = a – a cos θ , where a = OG = OG_o Then **GPE = m g a (1 – cos θ)**. θ_m is small , so $\cos\theta = 1 - \frac{\theta^2}{2}$, then GPE = $\frac{1}{2}$ m g a θ^2 ME = KE + GPE , then ME = $\frac{1}{2}$ I θ'^2 + $\frac{1}{2}$ m g a θ^2	1
	1-2		The pendulum oscillates without friction and air resistance is neglected, so the sum of works of non conservative forces is zero, then the mechanical energy of the system is conserved. ME = $\frac{1}{2}$ I θ'^2 + $\frac{1}{2}$ m g a θ^2 = constant , then $\frac{d\,ME}{dt} = 0$, thus $2(\frac{1}{2} I' \theta'') + 2(\frac{1}{2} mg\,a\,\theta') = 0 \Rightarrow \theta' (I \theta'' + mga\,\theta) = 0$. But θ′ = 0 is rejected, therefore: $\theta'' + \frac{m\,g\,a}{I}\theta = 0$ 2nd order differential equation in θ.	1
	1-3	1-3-1	$\theta = \theta_m \sin(\omega_o t + \varphi)$, then $\theta' = \omega_o \theta_m \cos(\omega_o t + \varphi)$ $\theta'' = -\omega_o^2 \theta_m \sin(\omega_o t + \varphi) = -\omega_o^2\,\theta$ Substitute θ″ in the differential equation: $-\omega_o^2\theta + \frac{m\,g\,a}{I}\theta = \theta(-\omega_o^2 + \frac{m\,g\,a}{I}) = 0$ θ = 0 is rejected , then $\omega_o^2 = \frac{m\,g\,a}{I}$, therefore $\omega o = \sqrt{\frac{m\,g\,a}{I}}$	0.75
		1-3-2	$T_o = \frac{2\pi}{\omega_o}$, then$T_o = 2\pi\sqrt{\frac{I}{mga}}$	0.5
	1-4	1-4-1	$T_o = \frac{25.2}{9}$, thus T_o = 2.8 s	0.5
		1-4-2	$T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{I}{mga}}$, then $T_o^2 = \frac{4\,\pi^2\,I}{m\,g\,a}$; $2.8^2 = \frac{4 \times 3.14^2 \times I}{2 \times 10 \times 0.24}$, therefore I = 0.95 kg.m^2	0.75
	1-5		M.E = $\frac{1}{2}$ I $\theta_m'^2$ = $\frac{1}{2}$ m g a θ_m^2 ; I $\times 0.36^2 = 2 \times 10 \times 0.24 \times 0.16^2$; I = 0.95 kg.m^2.	1
2	2-1		System: (Turntable - vase). External forces: the weight $M\vec{g}$ of the turntable ; the weight $m\vec{g}$ of the vase ; and the reaction $\vec{R}$ at the axle of rotation	0.5
	2-2		Moments relative to (Δ): $M_{\vec{R}} = M_{M\vec{g}} = 0$ since these forces are passing through the axis of rotation $M_{m\vec{g}} = 0$, since this force is parallel to the axis of rotation. $\sum M = M_{m\vec{g}} + M_{\vec{R}} + M_{M\vec{g}} = 0$. But $\sum M = \frac{d\,\sigma}{dt}$, then $\frac{d\,\sigma}{dt} = 0$. Therefore σ = constant..	1
	2-3		$I_t = \frac{1}{2} M R^2 = \frac{1}{2} \times 20 \times 0.5^2$ = 2.5 kg.m2 The angular momentum of the system is conserved, then $\sigma_{initial} = \sigma_{final}$ $I_t\,\theta'_t + 0 = (I' + I_t)\,\theta'_{system}$, so $2.5 \times 0.7 = (I' + 2.5)(0.45)$,then I′ = 1.39 kg.$m^2$	1

<u>Exercise 2</u> (7.5 points)

Part			Answer	Mark
1	1-1		Each energy level has a specific value , therefore the energy of the atom is quantized.	**0.5**
	1-2		$E_{ph} = \frac{hC}{\lambda}$; E_{ph} max if λ is minimum ; $E_{ph(max)} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{0.4 \times 10^{-6}} = 4.95 \times 10^{-19}$ J $= 3.093$ eV $E_{ph(min)} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{0.8 \times 10^{-6}} = 2.475 \times 10^{-19}$J $= 1.546$ eV	**0.5** **0.5**
	1-3		$W_{ion} = E_{\infty} - E_1 = 0 - (-5.14) = 5.14$ eV, $E_{ph(max)} = 3.093\text{eV} < W_{ion} = 5.14$ eV Therfore the white light cannot ionize the atom	**1**
	1-4		$E_{ph} = E_2 - E_1$, then $\frac{hC}{\lambda} = -3.04 + 5.14 = 2.1\text{eV} = 3.36 \times 10^{-19}$J $\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3.36 \times 10^{-19}} = 0.589 \times 10^{-6}\text{m} = 589$ nm.	**1**
2	2-1		$E_n = E_2 = -3.04$ eV since this photon excites the atom from E_1 to E_2 so it is emitted when the atom **<u>OR</u>** : $E_n - E_1 = E_{photon}$; $E_{photon} = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{589 \times 10^{-9} \times 1.6 \times 10^{-19}} = 2.1$ eV $E_n - \ = E_{photon} + E_1 = 2.1 - 5.14 = -3.04$ eV	**1**
	2-2	2-2-1	$E_{3/1} = E_3 - E_1 = 3.21$ eV $= 3.21 \times 10^{-6}$ MeV.	**0.75**
		2-2-2	$E_{3/1} = \Delta mc^2$ $\Delta m = \frac{3.21 \times 10^{-6}}{931.5} = 3.446 \times 10^{-9}$u.	**0.75**
	2-3	2-3-1	$P = P_1 + P_2$ But $P_1 = 2P_2$, then $P = 3P_2$, thus $P_2 = 2$W and $P_1 = 4$ W.	**0.5**
		2-3-2	$P_1 = \frac{nE_1}{t}$ then $n = \frac{t \times P_1}{E_1} = \frac{1 \times 4}{3.36 \times 10^{-19}} = 1.19 \times 10^{19}$ photons.	**1**

Exercise 3 (7 points) **Interference of light**

Part			Answer	Mark
1	1-1		At O, x = 0 , then $\delta_O = 0$, then O is the center of the central bright fringe.	0.5
	1-2		Dark fringe: $\delta = (2k+1)\frac{\lambda}{2}$, $k \in Z$, then $(2k+1)\frac{\lambda}{2} = \frac{a\,x}{D}$ thus $x = \frac{(2k+1)\lambda D}{2a}$	0.75
	1-3		$i = x_{K+1} - x_K = (2(k+1)+1)\frac{\lambda D}{2a} - (2k+1)\frac{\lambda D}{2a} = \frac{\lambda D}{a}$	0.5
	1-4	1-4-1	5 dark fringes	0.5
		1-4-2	AB = 5 i	0.5
		1-4-3	B is the center of the third dark fringe on the positive side of O.	0.5
		1-4-4	First dark fringe $x_1 = 1$ mm	0.5
	1-5		$x_1 = \frac{(2k+1)\lambda D}{2a}$, k = 0,then $D = \frac{2x_1}{\lambda}a = \frac{2 \times 1 \times 10^{-3}}{500 \times 10^{-9}}$ a , therefore D = 4000 a. **Or :** $x_B = \frac{(2k+1)\lambda D}{2a}$, k = 2 , then $D = \frac{2x_B}{5\lambda}a = \frac{2 \times 5 \times 10^{-3}}{5 \times 500 \times 10^{-9}}$ a , therefore D = 4000 a.	0.75
2	2-1		$\delta = SS_2P - SS_1P = (SS_2 - SS_1) + (S_2P - S_1P) = \frac{aZ}{d} + \frac{ax}{D}$.	0.5
	2-2		Central bright fringe : $\delta = 0$, then $0 = \frac{aZ}{d} + \frac{ax}{D}$. $x = -\frac{Z\,D}{d}$	0.5
	2-3		10th bright fringe , then : $x = -10i = -10\frac{\lambda D}{a} = -\frac{Z\,D}{d}$ $a = \frac{10\lambda d}{z} = 5\times10^{-4}$ m D = 4000a = 2m	1.5

Exercise 4 (7.5 points)

Characteristics of coil

Part			Answer	Mark
1	1-1		Self electromagnetic induction.	0.25
	1-2		Law of addition of voltage: $u_{MB} = u_{MN} + u_N$, then $ri + L\frac{di}{dt} + Ri = E$ At steady state: $i = I_0$= constant , thus $\frac{di}{dt} = 0$, therefore $I_0 = \frac{E}{r+R}$	0.75
	1-3	1-3-1	A t = 0 : i= 0 then $u_R = 0$, then E = $u_R + u_{coil}$ from graph E = 4.5 V.	0.5
		1-3-2	At steady state: $\frac{di}{dt} = 0$, then $u_{coil} = 0 + rI_0$; graphically : $u_{coil} \neq 0$ then ; $r \neq 0$	0.5
		1-3-3	rI_0= 1.5 V , then r = 15 Ω.	0.5
	1-4		$I_0 = \frac{E}{r + R_0}$, then $R_0 = -r + E/I_0$ = 30 Ω.	0.5
	1-5		$u_{MB} = u_{MN} + u_N$, thus $ri + L\frac{di}{dt} + Ri = E$; $(r + R) i + L\frac{di}{dt} = E$	0.5
	1-6	1-6-1	$\frac{di}{dt} = \frac{I_0}{\tau} e^{-\frac{t}{\tau}}, then\ E = (r + R_0)\left(I_0 - I_0 e^{-\frac{t}{\tau}}\right) + L\frac{I_0}{\tau} e^{-\frac{t}{\tau}}$ thus: $\tau = \frac{L}{r + R_0}$	0.75
		1-6-2	A t = τ : i = 0.63 I_0 = 0.063 A , then $u_R = Ri$ = 1.89 V	0.75
		1-6-3	$u_{coil} = E - u_R$ = 2.61 V	0.25
		1-6-4	Graphically τ = 1 ms	0.25
	1-7		L= $\tau(r + R_0)$ = 0.045H.	0.5
2	2-1		$E_{em} = 2.25 \times 10^{-6}$ J	0.25
	2-2		$\frac{1}{2} L I_0^2 = 2.25 \times 10^{-6}$, therefore L= 0.045H	0.5
	2-3		Slope = $-2.25 \times 10^{-4}/1.5 \times 10^{-3}$ = - 0.15 J/s	0.5
	2-4		Slope = $-rI_0^2$, therefore r = 15 Ω.	0.25