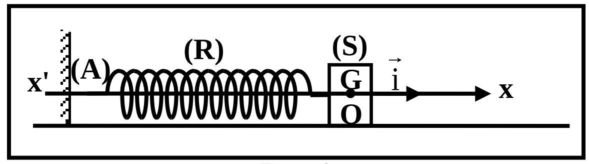
This exam is formed of four obligatory exercises in 4 pages.
The use of non-programmable calculator is recommended.

Exercise 1 (8 points) Mechanical oscillations

The aim of this exercise is to study the oscillation of a horizontal elastic pendulum. The pendulum is formed of:

- A block (S) of mass m;
- A massless horizontal spring (R) of stiffness k = 160 N/m.



Doc.1

We fix the spring (R) from its end (A) to a support. The other end is connected to (S).

(S) can slide on a horizontal rail and its center of mass (G) can move along a horizontal x-axis of unit vector $\vec{i}$.
At equilibrium, (G) coincides with the origin O of the x-axis (Doc.1).
The horizontal plane containing (G) is taken as a gravitational potential energy reference.
Take $\pi^2 = 10$.

1- Free undamped oscillations

At the instant $t_0 = 0$, (S) is shifted to the left by a displacement $x_0 = -2\sqrt{2}$ cm and then it is launched with an initial velocity $\vec{v_0} = v_0\,\vec{i}$, where $v_0 < 0$. (S) oscillates without friction with an amplitude $X_m = 4$ cm and a proper period $T_0 = 0.35$ s.

At an instant t, the abscissa of (G) is $x = \overline{OG}$ and the algebraic value of its velocity is $v = \frac{dx}{dt}$.

1.1) Calculate the mechanical energy of the system [(S) - spring - Earth].

1.2) Derive the second order differential equation in x that governs the motion of (G).

1.3) The solution of this differential equation is of the form $x = X_m \cos\left(\frac{2\pi}{T_0}t + \varphi\right)$, where φ is constant.

1.3.1) Determine the expression of the proper period T_0 in terms of m and k.

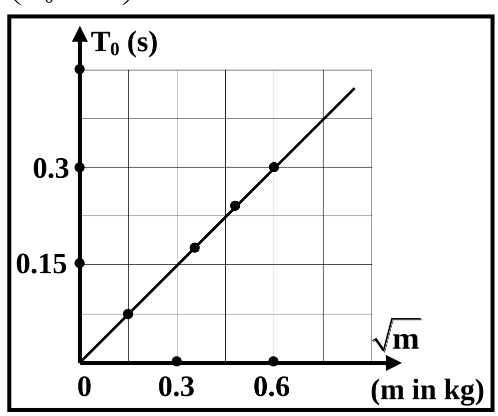
1.3.2) Deduce the value of m.

1.3.3) Determine the value of φ.

1.4) Using the principle of conservation of the mechanical energy, show that $\left(\frac{T_0}{2\pi}\right)^2 v_0^2 = X_m^2 - x_0^2$.

1.5) Deduce the value of v_0.

1.6) In order to verify the value of the stiffness k, we repeat the above experiment by attaching successively blocks of different masses to the spring. We measure for each mass the corresponding value of the proper period. An appropriate device plots the graph of T_0 versus $\sqrt{m}$ (Doc. 2).



Doc. 2

1.6.1) Determine the expression of T_0 as a function of $\sqrt{m}$, using document 2.

1.6.2) Deduce the value of k.

2- Forced oscillations

Friction is no longer neglected. End (A) of the spring is now attached to a vibrator of adjustable frequency "f " vibrating along the axis of the spring. We notice that the amplitude of oscillation of (S) varies with "f "; the amplitude attains its maximum value for a frequency $f_1 = 2.86$ Hz.

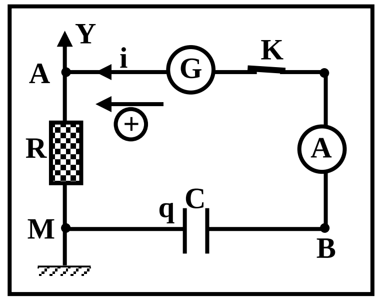
2.1) Name the exciter and the resonator.

2.2) Name the physical phenomenon that takes place for $f = f_1$.

2.3) Deduce again the value of k.

<u>Exercise 2</u> (8 points) Determination of the capacitance of a capacitor

The aim of this exercise is to determine, by two different methods, the capacitance C of a capacitor. For this aim, we consider: a capacitor of capacitance C initially uncharged, a resistor of resistance R, a switch K, an ammeter (A) of negligible resistance and a generator (G).



Doc. 3

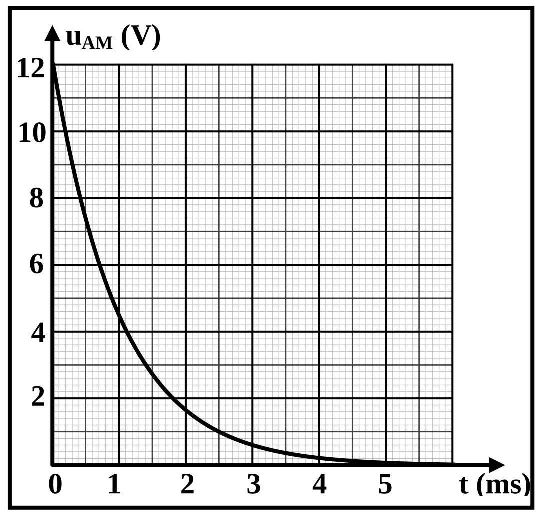
1. First experiment

(G) provides a constant voltage $u_{AB} = E = 12$ V.

We connect in series the capacitor, the resistor and the ammeter (A) across the terminals of (G) (Doc. 3).

At the instant $t_0 = 0$, we close K, thus the circuit carries a current i and the ammeter indicates a value $I_0 = 0.012$ A.

An oscilloscope is used to display the variation of the voltage u_{AM} across the resistor as a function of time (Doc. 4).



Doc. 4

1.1) Derive the differential equation that describes the variation of the voltage $u_C = u_{MB}$.

1.2) Deduce that the differential equation in i is: $i + RC\dfrac{di}{dt} = 0$.

1.3) The solution of this differential equation is of the form:

$i = I_0\, e^{\frac{-t}{\tau}}$, where I_0 and τ are constants.

Show that $I_0 = \dfrac{E}{R}$ and $\tau = RC$.

1.4) Using document 4:

1.4.1) show that the value of R is 1 kΩ ;

1.4.2) determine the value of τ ;

1.4.3) deduce the value of C.

2. Second experiment

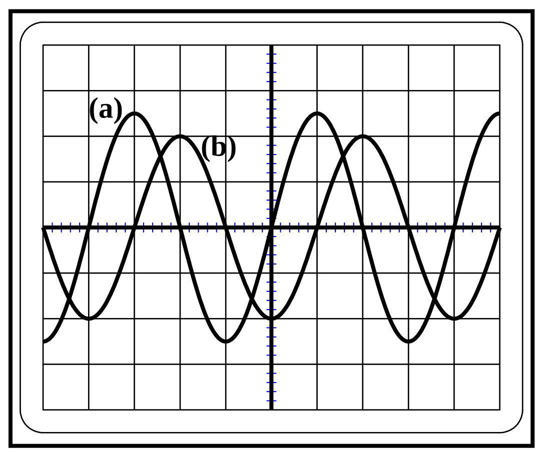
(G) provides an alternating sinusoidal voltage. An oscilloscope is connected in the circuit in order to display the voltages u_{AM} on channel (Y_1) and u_{MB} on channel (Y_2) [the "INV" button being pressed].

Document 5 shows the curves of the voltages u_{AM} and u_{MB}.

Take: $\pi = 3.125$.

The adjustments of the oscilloscope are:

- horizontal sensitivity: 2.5 ms/div;
- vertical sensitivity: 5 V/div on channel (Y_1);
  10 V/div on channel (Y_2).



Doc. 5

2.1) Waveform (b) represents the voltage u_{MB}. Why?

2.2) Calculate the period of the voltage provided by (G) and deduce the angular frequency ω.

2.3) Calculate the maximum value of the voltages u_{AM} and u_{MB}.

2.4) Calculate the phase difference φ between the voltage u_{MB} and the current i.

2.5) Knowing that the current i is given by: $i = I_m \cos(\omega t)$.

2.5.1) Determine the expressions of u_{AM} and u_{MB} as a function of time t;

2.5.2) Calculate the value of I_m.

2.6) Deduce the value of C.

<u>Exercise 3</u> (7 points) Determination of the age of a liquid

Tritium $^{3}_{1}H$ is a radioactive hydrogen isotope. Tritium is produced in the upper atmosphere by cosmic rays and brought to Earth by rain. The tritium can be used to determine the age of liquids containing this isotope of hydrogen.
In this exercise, we intend to determine the age of a liquid in an old bottle using the variation in the activity of tritium.

1. Radioactive decay of tritium

Tritium is a beta-minus (β^-) emitter. It decays into one of the isotopes of helium without the emission of gamma radiation.

1.1) Complete the equation of the decay of tritium and determine A and Z.

$$^{3}_{1}H \rightarrow {}^{A}_{Z}He + \ldots$$

1.2) The helium nucleus is produced in the ground state. Why?

1.3) A particle X accompanies the above disintegration in order to satisfy a certain law.
Name this particle and this law.

2. Determination of the radioactive period of tritium

Consider a sample of the radioactive isotope tritium $^{3}_{1}H$.
At an instant $t_0 = 0$, the number of nuclei in this sample is N_0.
The activity A of the radioactive sample represents the number of disintegrations per unit time.
The activity at an instant t is given by the following expression: $A = -\frac{dN}{dt}$, where N is the number of the remaining (undecayed) nuclei at the instant t.

2.1) Show that the first order differential equation that governs the variation of N is: $\frac{dN}{dt} + \lambda N = 0$, where λ is the decay constant of the radioactive isotope.

2.2) Verify that $N = N_0\, e^{-\frac{t}{\tau}}$ is a solution of the above differential equation, where $\tau = \frac{1}{\lambda}$.

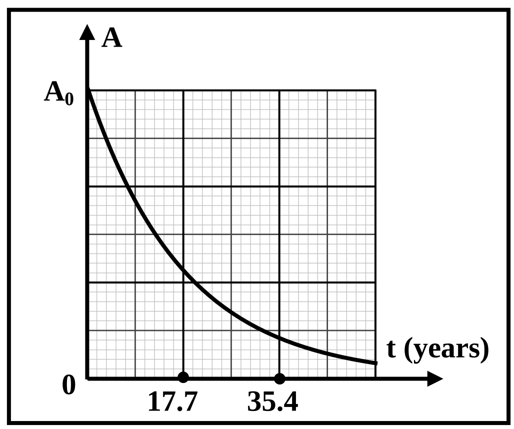
2.3) Deduce that the expression of the activity is given by:
$A = A_0\, e^{-\frac{t}{\tau}}$, where A_0 is the initial activity of the sample.

2.4) Calculate A in terms of A_0 when $t = \tau$.

2.5) Document 6 represents the activity of a sample of tritium as a function of time.

2.5.1) Show that $\tau = 17.7$ years.

2.5.2) Deduce the radioactive period of tritium.



Doc. 6

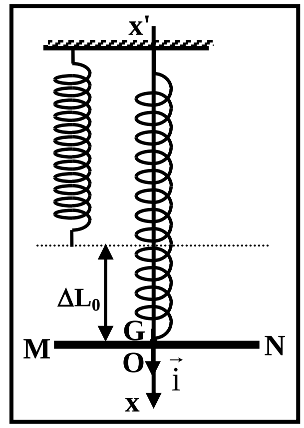
3. Determination of the age of a liquid

An old bottle containing a certain liquid is just opened (in 2018). It is found that the activity of tritium in this liquid is 10.4 % of the initial activity of the same liquid freshly prepared. Determine the year of production of the liquid in the old bottle.

<u>Exercise 4</u> (7 points) **Electromagnetic induction**

The aim of this exercise is to determine the magnitude B of a uniform magnetic field $\vec{B}$.

Consider a spring of stiffness k and of negligible mass attached from its upper end to a fixed support. Its lower end is attached to a copper rod MN of mass m and length ℓ. At equilibrium, the elongation of the spring is ΔL_0 and the center of mass G of the rod coincides with the origin O of a vertical x-axis of unit vector $\vec{i}$ (Doc.7).



Doc. 7

1. **Rod in equilibrium**

1.1) Name the external forces acting on the rod at the equilibrium position.

1.2) Determine the relation among m, g, k and ΔL_0.

2. **Electromagnetic induction**

The rod MN may slide without friction along two vertical metallic rails (PP') and (QQ'). During sliding the rod remains perpendicular to the two rails.

The two rails are separated by a distance ℓ and a capacitor, initially uncharged, of capacitance C is connected between P and Q.

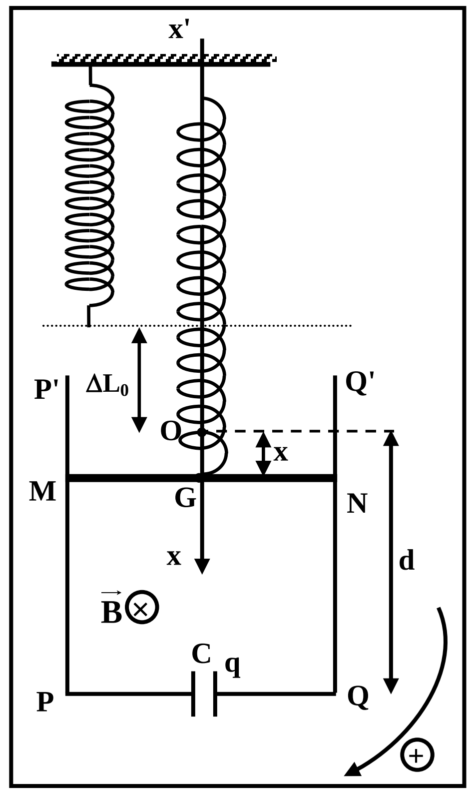
Neglect the resistance of the rod and of the rails.

This set-up is placed in the region of a horizontal uniform magnetic field $\vec{B}$ perpendicular to the plane of the rails.

At the equilibrium position G is found at a distance d from (PQ).

The rod is pulled vertically downwards from its equilibrium position by a distance X_m, and then it is released without initial velocity, thus G oscillates about its equilibrium position O.

At an instant t, G is defined by its abscissa $x = \overline{OG}$ and the algebraic value of its velocity is $v = \frac{dx}{dt}$ (Doc. 8).



Doc. 8

2.1) Taking the positive sense, shown in document 8, into consideration , show that the expression of the magnetic flux crossing the area MNQP is given by $\varphi = B\ell d - B\ell x$.

2.2) Deduce the expression of the electromotive force "e" induced in the rod in terms of B, ℓ and v.

2.3) Knowing that $u_{QP} = u_C = e$, show that the expression of the current induced in the circuit MNQP is $i = C\,B\,\ell\,\frac{dv}{dt}$.

3. **Free oscillations**

The rod is subjected to an electromagnetic force (Laplace's force)

$\vec{F} = -B^2\,\ell^2\,C\,\frac{dv}{dt}\,\vec{i}$.

3.1) Applying Newton's second law $\Sigma\overrightarrow{F_{ext}} = m\,x''\,\vec{i}$, show that the second order differential equation that governs the variation of the abscissa x is given by $x'' + \frac{k}{m + B^2\ell^2 C}\,x = 0$.

3.2) Specify the nature of motion of the rod.

3.3) Deduce the expression of the proper period T_0 of oscillation of the rod.

3.4) The duration of 10 oscillations is 4.69 s. Determine the value of B, knowing that m = 10 g, ℓ = 10 cm, C = 8 mF and k = 1.8 N/m.

وزارة التربية والتعليم العالي	امتحانات الشهادة الثانوية العامة	دورة العام ٢٠١٨ الاستثنائيّة
المديرية العامة للتربية	الفرع: علوم عامة	الإثنين في ٦ آب ٢٠١٨
دائرة الامتحانات الرسميّة	مسابقة في مادة الفيزياء	

أسس التصحيح - إنكليزي

Exercise 1 (8 points) Mechanical oscillations

Part			Answer	Marks
1	1.1		$ME = \frac{1}{2} k X_m^2 = \frac{1}{2}(160)(4\times10^{-2})^2 = 0.128 J$	0.5
	1.2		$ME = KE + PE_e = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$. Friction is neglected, so sum of works of nonconservative forces is zero, then ME is conserved. $\frac{d(ME)}{dt} = 0 = mvv' + kxx'$, so $x'(mx'' + kx) = 0$, thus $x'' + \frac{k}{m} x = 0$	0.75
	1.3	1.3.1	$x = X_m \cos\left(\frac{2\pi}{T_0} t + \varphi\right)$; $x' = -X_m \frac{2\pi}{T_0} \sin\left(\frac{2\pi}{T_0} t + \varphi\right)$; $x'' = -X_m \left(\frac{2\pi}{T_0}\right)^2 \cos\left(\frac{2\pi}{T_0} t + \varphi\right)$; Substituting in the differential equation gives : $-X_m \left(\frac{2\pi}{T_0}\right)^2 \cos\left(\frac{2\pi}{T_0} t + \varphi\right) + \frac{k}{m} X_m \cos\left(\frac{2\pi}{T_0} t + \varphi\right) = 0$; $\left(\frac{2\pi}{T_0}\right)^2 = \frac{k}{m}$, so $T_0 = 2\pi\sqrt{\frac{m}{k}}$	1
		1.3.2	$T_0 = 2\pi\sqrt{\frac{m}{k}}$, so $m = \frac{T_0^2 . k}{4\pi^2}$, then $m = 0.49 kg = 490 g$	0.75
		1.3.3	$x_0 = x_m \cos\varphi$; $-2\sqrt{2} = 4\cos\varphi$; $\cos\varphi = -\frac{\sqrt{2}}{2}$; so $\varphi = \frac{3\pi}{4}$ rad or $\varphi = -\frac{3\pi}{4}$ rad. But, $v_0 = -X_m \frac{2\pi}{T_0} \sin\varphi$. Since $v_0 < 0$ then $\sin\varphi > 0$; therefore, $\varphi = 3\pi/4$ rad	1
	1.4		$ME\|_{x_0} = ME\|_{x_m}$, then $\frac{1}{2} m v_0^2 + \frac{1}{2} k x_0^2 = \frac{1}{2} k X_m^2$, then $mv_0^2 = k(X_m^2 - x_0^2)$. Substituting $m = \frac{kT_0^2}{4\pi^2}$ into the last expression gives : $\left(\frac{T_0}{2\pi}\right)^2 v_0^2 = X_m^2 - x_0^2$	0.75
	1.5		$v_0^2 = \frac{(X_m^2 - x_0^2)4\pi^2}{T_0^2}$, then $v_0 = 0.511$ m/s	0.5
	1.6	1.6.1	T_0 is proportional to $\sqrt{m}$ then $T_0 = \text{slope}\sqrt{m}$. slope $= \frac{\Delta T_0}{\Delta\sqrt{m}} = \frac{0.3}{0.6} = 0.5\ s/\sqrt{kg}$ Then, $T_0 = 0.5 \times \sqrt{m}$ (S. I.)	1
		1.6.2	$T_0 = \frac{2\pi}{\sqrt{k}}\sqrt{m}$, then slope $= \frac{2\pi}{\sqrt{k}} = 0.5$, so $k = \frac{4\pi^2}{0.25} = \frac{4\times10}{0.25} = 160 N/m$	0.75
2	2.1		Exciter : the vibrator ; Resonator : the oscillator	0.5
	2.2	2.2.1	Amplitude resonance	0.25
		2.2.2	At resonance : $f_1 \cong f_0 = 2,86$ Hz. Replacing $T_0 = \frac{1}{2,86}$ into $T_0 = 2\pi\sqrt{\frac{m}{k}}$, gives $k \cong 160$ N/m	0.25

Exercise 2 (8 points) **Determination of the capacitance of a capacitor**

Part			Answer	Marks
1	1.1		$u_{AB} = u_{AM} + u_{MB}$, then $E = u_C + Ri$ and $i = dq/dt = C\, du_c/dt$ Then : $E = u_C + RC\, du_c/dt$	0.5
	1.2		Differentiating the last equation with respect to time gives : $0 = du_c/dt + RC\, d^2u_c/dt^2$, but $i = C\, du_c/dt$ and $\frac{di}{dt} = C\, d^2u_c/dt^2$, then : $0 = \frac{i}{C} + R\frac{di}{dt}$, then $0 = i + RC\frac{di}{dt}$	0.75
	1.3		$\frac{di}{dt} = -\frac{I_0}{\tau} e^{-t/\tau}$, substituting in the differential equation gives : $0 = I_0 e^{-t/\tau} + RC\,(\frac{-I_0}{\tau})\, e^{-t/\tau}$ $0 = I_0\,(1 - \frac{RC}{\tau})\, e^{-t/\tau}$ for each value of t, then $\tau = RC$ At $t = 0$: $i = I_0 e^0 = I_0$ and At $t = 0$: $i = \frac{E}{R}$, then $\frac{E}{R} = I_0$	1
	1.4	1.4.1	$I_o = \frac{E}{R}$ then $R = \frac{12}{0.012} = 1000\ \Omega$	0.5
		1.4.2	At $t = \tau$: $u_R = 0.37\, E = 0,37 \times 12 = 4.44$ V Graphically: $\tau = 1$ ms $= 10^{-3}$ s	0.5
		1.4.3	$C = \frac{\tau}{R} = \frac{10^{-3}}{10^3} = 10^{-6}$ F = 1 µF	0.5
2	2.1		In a series R-C circuit, i leads u_C . Curve of i is similar to that of u_R, then u_R leads u_C. Since curve (a) leads curve (b), then curve (b) represents u_C	0.5
	2.2		$T = 4 \times 2.5 = 10$ ms $\omega = \frac{2\pi}{T} = \frac{2\pi}{10-2} = 200\,\pi$ rad/s = 625 rad/s	0.75
	2.3		$(u_{AM})_m = 2.5 \times 5 = 12.5$ v $(U_{MB})_m = 2 \times 10 = 20$ V	0.5
	2.4		$\varphi = \frac{2\pi d}{D} = \frac{2\pi \times 1}{4} = \frac{\pi}{2}$ rad	0.5
	2.5	2.5.1	$u_R = u_{AM} = 12.5 \cos(200\,\pi t)$ S.I. u_C lags behind u_R by $\pi/2$ rad, then : $u_C = u_{MB} = 20\, \cos(200\,\pi t - \pi/2)$ S.I.	0.5
		2.5.2	$I_m = \frac{(U_{AM})_m}{R} = \frac{12.5}{10^3} = 12.5 \times 10^{-3}$ A	0.5
	2.6		$i = C\frac{du_C}{dt}$, then $i = C \times 20 \times [-200\pi \sin(200\pi t - \frac{\pi}{2})]$ and $i = I_m \cos(\omega t)$ $I_m \cos(\omega t) = - C \times 4 \times 10^3\, \pi\, \sin(200\pi\, t - \frac{\pi}{2})$ $I_m \cos(\omega t) = C \times 4 \times 10^3\, \pi \cos(200\pi\, t)$, so $I_m = C \times 4 \times 10^3\, \pi$ Then: $12.5 \times 10^{-3} = C \times 4 \times 10^3 \times \pi$ So: $C = \frac{12.5 \times 10^{-3}}{4 \times 10^3 \times \pi} = \frac{12.5 \times 10^{-6}}{2 \times 6.25} = 10^{-6}$ F = 1 µF	1

Exercise 3 (7 points) **Determination of the age of a liquid**

<table>
<tr><th colspan="3">Part</th><th>Answer</th><th>Marks</th></tr>
<tr><td rowspan="3">1</td><td colspan="2">1.1</td><td>$^{3}_{1}H \rightarrow {}^{A}_{Z}He + {}^{0}_{-1}e + {}^{0}_{0}\nu^{-}$
Applying the law of conservation of mass number: 3 = A + 0 , then A = 3
Applying the law of conservation of charge number: 1 = z -1 , then z = 2</td><td>1</td></tr>
<tr><td colspan="2">1.2</td><td>Tritium decays into one of the isotopes of helium without the emission of gamma radiation; therefore, the helium nucleus is produced in the ground state.</td><td>0.5</td></tr>
<tr><td colspan="2">1.3</td><td>The particle is the antineutrino.
The law is: the law of conservation of total energy (or conservation of energy)</td><td>0.5</td></tr>
<tr><td rowspan="6">2</td><td colspan="2">2.1</td><td>$A = -\frac{dN}{dt} = \lambda N$, then $\frac{dN}{dt} + \lambda N = 0$</td><td>0.5</td></tr>
<tr><td colspan="2">2.2</td><td>$N = N_o\, e^{-\frac{t}{\tau}}$, but $\frac{dN}{dt} = -\frac{N_o}{\tau} e^{-\frac{t}{\tau}} = -\frac{N}{\tau}$.
Substituting in the differential equation, gives: $-\frac{N}{\tau} + \lambda N = 0$
But $\tau = \frac{1}{\lambda}$, then: $-\lambda N + \lambda N = 0$</td><td>0.75</td></tr>
<tr><td colspan="2">2.3</td><td>$A = \lambda N = \lambda N_o\, e^{-\frac{t}{\tau}}$, but $A_o = \lambda N_o$; therefore, $A = A_o\, e^{-\frac{t}{\tau}}$</td><td>0.75</td></tr>
<tr><td colspan="2">2.4</td><td>$A = A_o\, e^{-\frac{\tau}{\tau}} = A_o\, e^{-1}$, therefore $A = 0.37\, A_o$</td><td>0.75</td></tr>
<tr><td rowspan="2">2.5</td><td>2.5.1</td><td>At $t = \tau$, $A = 0.37\, A_o$.
Graphically, when $A = 0.37\, A_o$; $t = \tau = 17.7$ years.</td><td>0.5</td></tr>
<tr><td>2.5.2</td><td>$\tau = \frac{1}{\lambda} = \frac{T}{\ell n\ 2}$, then $T = \tau\ \ell n\ 2 = 17.7\ (\ell n 2)$, therefore T = 12.3 years.</td><td>0.75</td></tr>
<tr><td colspan="3">3</td><td>$A = A_o\, e^{-\frac{t}{\tau}}$,then $0.104\, A_o = A_o\, e^{-\frac{t}{\tau}}$, so $\ell n\,(0.104) = -t/\tau$
$t = -\ell n\,(0.104)\,(17.7) \cong 40$ years. Year of production = 2018 – 40 = 1978.</td><td>1</td></tr>
</table>

Exercise 4 (7 points) **Electromagnetic induction**

Part		Answer	Marks
1	1.1	The weight $\vec{W} = m\vec{g}$ and the spring force $\vec{T}$	0.5
	1.2	The rod is at equilibrium: $m\vec{g} + \vec{T} = \vec{0}$, so $\vec{T} = -m\vec{g}$, then $k\Delta L_0 = mg$	1
2	2.1	$\phi = BS\cos(\vec{n},\vec{B}) = B(d-x)\ell\cos 0 = Bd\ell - B\ell x$	0.75
	2.2	$e = -\frac{d\varphi}{dt} = B\ell v$	0.5
	2.3	$i = \frac{dq}{dt} = C\frac{du_C}{dt} = C\frac{de}{dt} = CB\ell\frac{dv}{dt}$	0.75
3	3.1	$m\vec{g} + \vec{T} + \vec{F} = mx''\vec{i}$; then : $mg - k(\Delta L_0 + x) - B^2\ell^2 C\frac{dv}{dt} = mx''$, $k\Delta L_0 = mg$; Then : $x'' + \frac{k}{m + B^2\ell^2 C}x = 0$	1.25
	3.2	The differential equation is of the form: $x'' + \omega_0^2 x = 0$ with ω_0 being a positive constant, then it is a simple harmonic motion.	0.5
	3.3	$\omega_0^2 = \frac{k}{m + B^2\ell^2 C}$; The proper period $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m + B^2\ell^2 C}{k}}$	0.75
	3.5	$T_0 = \frac{4.69}{10} = 0.469\,s$; $T_0^2 k = 4\pi^2(m + B^2\ell^2 C)$ $B^2 = \frac{1}{\ell^2 C}\left[\frac{T_0^2 k}{4\pi^2} - m\right]$ substituting the data in this expression gives B = 0.7 T B = 0.699 T (if π = 3,14) B = 0.6 T (if we substitute the precise value of π)	1