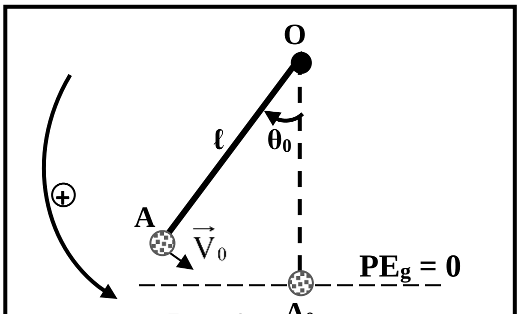
مسابقة في مادة الفيزياء — الاسم:
المدة: ساعتان ونصف — الرقم:

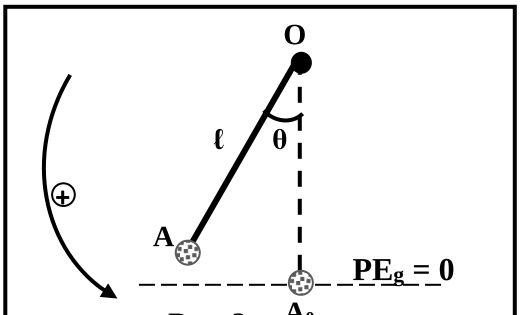
<u>This exam is formed of four obligatory exercises in four pages.</u>
<u>The use of non-programmable calculator is recommended.</u>

Exercise 1 (5 pts) Simple pendulum

A simple pendulum consists of a particle of mass m = 50 g attached from the lower end A of a massless and inextensible string OA of length ℓ.
This pendulum may oscillate in the vertical plane about a horizontal axis (Δ) passing through the upper extremity O of the string.
The pendulum is shifted in the negative direction from its equilibrium position. At an instant $t_0 = 0$, the angular abscissa of the pendulum is $\theta_0 = -\frac{\pi}{36}$ rad, and the particle is launched in the positive direction with a velocity $\vec{V}_0$ of magnitude V_0 (Doc. 1).
At an instant t, the angular abscissa of the pendulum is θ and the speed of the particle is $v = \ell\,|\theta'| = \ell\left|\frac{d\theta}{dt}\right|$ (Doc. 2).



Doc. 1



Doc. 2

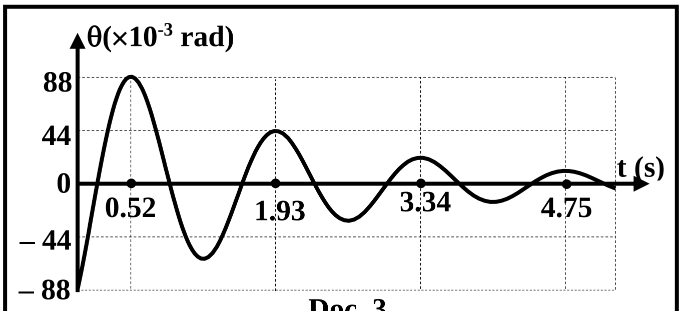
Take:
- the horizontal plane containing A_0, the position of A at equilibrium, as the reference level for gravitational potential energy;
- $g = 10\ m/s^2$.

1) Suppose that the pendulum oscillates without friction.
The second order differential equation in θ that describes the motion of the pendulum is:

$$\theta'' + 20\,\theta = 0 \quad \text{(SI)}.$$

1.1) The pendulum performs simple harmonic motion. Justify.
1.2) Calculate the value of the proper (natural) period T_0 of the pendulum.
1.3) Knowing that the proper period of the pendulum is $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$, show that $\ell = 50$ cm.
1.4) The mechanical energy of the system (Pendulum, Earth) at an instant t is ME.
1.4.1) Show that the expression of the mechanical energy is $ME = \frac{1}{2} m v^2 + m g \ell (1 - \cos\theta)$.
1.4.2) Deduce the value of V_0 knowing that $ME_0 = 1.95 \times 10^{-3}$ J at $t_0 = 0$.

2) In reality the pendulum is submitted to a force of friction. We repeat the above experiment and an appropriate device shows the angular abscissa θ of the pendulum as a function of time (Doc. 3).
Using document 3:
2.1) indicate the type of oscillations;
2.2) calculate the mechanical energy of the system (Pendulum, Earth) at t = 0.52 s;
2.3) deduce the average power lost by the system (Pendulum, Earth) between $t_0 = 0$ and t = 0.52 s.

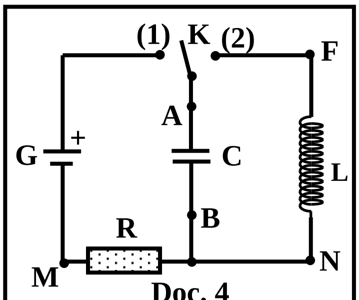


Doc. 3

Exercise 2 (5 pts) Characteristics of electric components

The aim of this exercise is to determine the capacitance C of a capacitor and the inductance L of a coil. For this purpose, we connect the circuit of document 4 which includes:

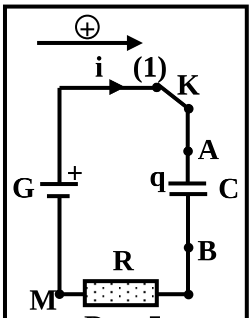
- an ideal battery G of electromotive force E = 2 V;
- a resistor of resistance R = 1 kΩ;
- a capacitor of capacitance C;
- a coil of inductance L and negligible resistance;
- a switch K.



Doc. 4

1) Series (R-C) circuit

The capacitor is initially uncharged. At the instant $t_0 = 0$, we turn K to position (1). At an instant t, the charge of plate A is q and the current in the circuit is i (Doc. 5).



Doc. 5

1.1) Name the physical phenomenon that takes place in the circuit.

1.2) Show that the differential equation that governs the variation of the voltage $u_{AB} = u_C$ across the capacitor is: $\tau \frac{du_C}{dt} + u_C = E$, where $\tau = RC$ is the time constant of the circuit.

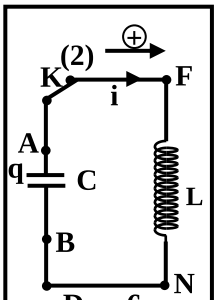
1.3) $u_C = 2\,(1 - e^{-1000\,t})$ (u_C in V and t in s) is a solution of this differential equation. Determine the value of τ.

1.4) Deduce the value of C.

2) (L-C) circuit

The capacitor is fully charged. At an instant $t_0 = 0$, taken as a new initial time, we turn K to position (2).

At an instant t, the charge of plate A is q and the current in the circuit is i (Doc.6).



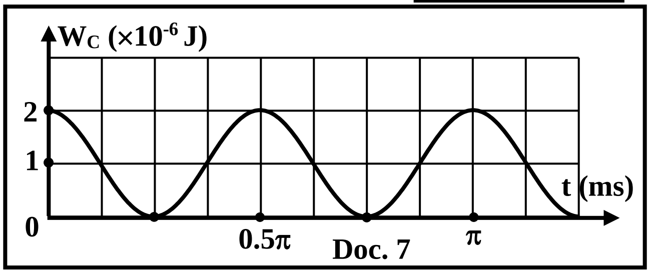
Doc. 6

2.1) Derive the differential equation that governs the variation of the charge q.

2.2) Deduce that the expression of the proper (natural) period T_0 of the circuit is $T_0 = 2\pi\sqrt{LC}$.

2.3) The curve of document 7 represents the electric energy W_C stored in the capacitor as a function of time.
Determine the value of T_0 knowing that $T_0 = 2T_E$, where T_E is the period of the electric energy.

2.4) Deduce the value of L.



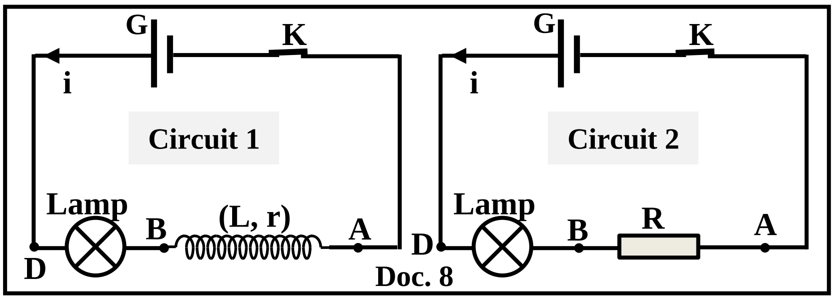
Doc. 7

Exercise 3 (5 pts) Self induction

We consider a coil of inductance L and resistance r, a resistor of resistance R = 8 Ω, a switch K, an incandescent lamp and an ideal battery (G) of electromotive force E = 10 V.
The aim of this exercise is to study the effect of the coil on the brightness of the lamp in a DC series circuit, and to determine its characteristics.

1) Brightness of the lamp

We set up successively circuit 1 and circuit 2 of document 8.
Statements 1 and 2 below describe the brightness of the lamp after closing K.



Doc. 8

__**Statement 1**__: The lamp glows instantly at the instant of closing the switch.
__**Statement 2**__: After closing the switch, the brightness of the lamp increases gradually and becomes stable after a certain time.

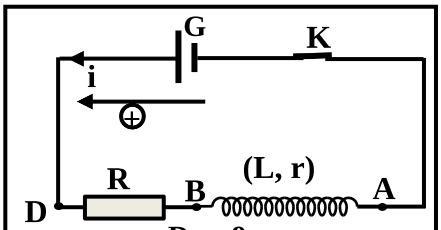
Match each statement to the convenient circuit.

2) Determination of L and r

We connect the coil and the resistor in series across (G) as shown in document 9.

At the instant $t_0 = 0$, K is closed.

At an instant t, the circuit carries a current i.



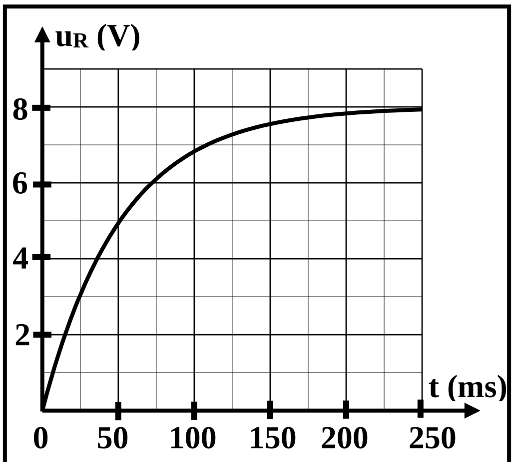
Doc. 9

2.1) Prove, by applying the law of addition of voltages, that the differential equation that describes the variation of the voltage $u_{DB} = u_R$ is: $\frac{L}{R}\frac{du_R}{dt} + \left(\frac{R+r}{R}\right)u_R = E$.

2.2) Deduce that the expression of the voltage across the resistor in the steady state is: $U_{Rmax} = E\frac{R}{R+r}$.

2.3) The solution of this differential equation is $u_R = U_{Rmax}(1 - e^{\frac{-t}{\tau}})$, where $\tau = \frac{L}{R+r}$.

A convenient apparatus draws u_R as a function of time (Doc. 10).



Doc. 10

2.3.1) Use document 10 to indicate the value of U_{Rmax}.

2.3.2) Determine the value of r.

2.3.3) Use document 10 to determine the value of τ.

2.3.4) Deduce the value of L.

Exercise 4 (5 pts) Stray bullets

The aim of this exercise is to determine the thermal energy produced during the motion of a bullet fired from a rifle and to show its danger.

A bullet (S) taken as a particle of mass $m = 7\times10^{-3}$ kg is fired from point O on the ground with an initial velocity $\vec{V}_0 = V_0\,\vec{j}$. During the whole motion, the bullet is submitted to air resistance.

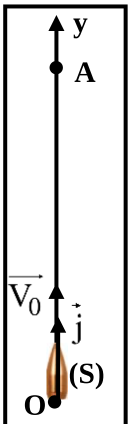
Take:

- $g = 10\ m/s^2$;
- the horizontal plane containing O as a reference level for gravitational potential energy.

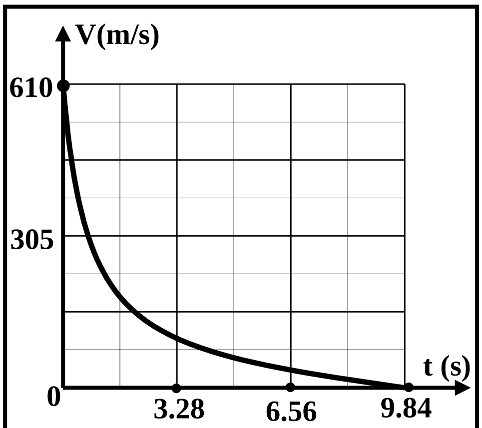
1) Upward motion of the bullet

The bullet (S) is fired vertically upward from point O at an instant $t_0 = 0$. (S) moves along the y-axis of origin O oriented positively upward. (S) reaches point A of maximum height h at $t_1 = 9.84$ s (Doc.11).

The graph of document 12 represents the speed V of (S) as a function of time during its upward motion between O and A.



Doc. 11



Doc. 12

1.1) Determine, using document 12, the linear momenta $\vec{P}_0$ and $\vec{P}_1$ of (S) at $t_0 = 0$ and at $t_1 = 9.84$ s respectively.

1.2) Deduce the variation in the linear momentum $\Delta\vec{P}$ of (S) between t_0 and t_1.

1.3) Given that $m\vec{g} + \vec{f} = \frac{\Delta\vec{P}}{\Delta t}$, where $\Delta t = t_1 - t_0$ and $\vec{f}$ is the average friction force acting on (S) during Δt. Prove that the magnitude of $\vec{f}$ is $f \cong 0.364$ N.

1.4) Calculate the mechanical energy ME_0 of the system [(S)-Earth] at $t_0 = 0$.

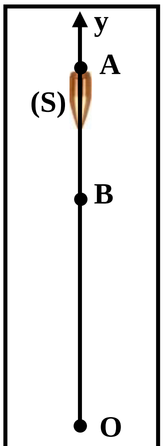
1.5) Given that $\Delta ME = -f \times h$, where ΔME is the variation in the mechanical energy of the system [(S)-Earth] during $\Delta t = t_1 - t_0$. Prove that $h \cong 3000$ m.

1.6) Deduce the value of the thermal energy W_{th1} produced during the upward motion of (S) knowing that $W_{th1} = |\Delta ME|$.

2) Downward motion of the bullet

Assume that the trajectory of (S) remains vertical.

(S) starts its downward motion from point A and passes through point B (AB = 352 m) and reaches the ground at O with a speed V = 44 m/s (Doc. 13). The magnitude of the friction force acting on (S) during its motion between B and O is $f_1 = 0.07$ N.



Doc. 13

2.1) Determine the value of the thermal energy W_{th2} produced during the motion of (S) between B and O knowing that $W_{th2} = |W_{\vec{f}_1}|$.

2.2) Calculate the thermal energy produced during the downward motion of (S) between A and O, knowing that the thermal energy produced during the downward motion of (S) between A and B is 18 J.

3) Danger of the stray bullet

The bullet can penetrate the skin of a human if its speed exceeds 61 m/s.

A bullet (S') identical to (S) is fired upward at a slight angle from the vertical (around 15^0), it follows a curvilinear path and reaches the ground at a speed 90 m/s.

Specify whether (S) or (S') is more dangerous when hitting a human as it reaches the ground.

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Exercise 1 (5 pts) Simple pendulum

Part			Answer	Mark
1	1.1		The differential equation $\theta'' + 20\,\theta = 0$ is of the form: $\theta'' + \omega_0^2\,\theta = 0$, then it is a simple harmonic motion.	0.25
	1.2		$\omega_0 = \sqrt{20}$ rad/s $T_0 = \frac{2\pi}{\omega_0} = \frac{2\,\pi}{\sqrt{20}}$, then $T_0 = 1.405$ s	0.75
	1.3		$T_0 = 2\pi\sqrt{\frac{\ell}{g}}$, then $1.405 = 2\pi\sqrt{\frac{\ell}{10}}$, hence $1.974 = 4\pi^2\,\frac{\ell}{10}$, so $\ell = 0.5$ m	0.5
	1.4	1	ME = KE + GPE But, GPE = m g z = m g ($\ell - \ell\cos\theta$) = m g ℓ ($1 - \cos\theta$) (Figure) $KE = \frac{1}{2} I\,\theta'^2$, but $I = m\,\ell^2$ and $v = \ell\,\theta'$ Then, $ME = \frac{1}{2}\,m\,v^2 + mg\ell\,(1 - \cos\theta)$	1
		2	$ME_o = \frac{1}{2}\,m\,V_0^2 + mg\,\ell(1 - \cos\theta_0)$ $1.95 \times 10^{-3} = \frac{1}{2} \times 0.05\,V_0^2 + 0.05 \times 10 \times 0.5\,[1 - \cos(\frac{-\pi}{36})]$, then $V_0 = 0.2$ m/s	0.75
2	2.1		Free undamped mechanical oscillations	0.25
	2.2		$ME_{(0.52)} = 0 + mg\ell(1 - \cos\theta_{(0.52)}) = 0.05 \times 10 \times 0.5\,[1 - \cos(88\times10^{-3})]$ Then, $ME_{(0.52)} = 9.67 \times 10^{-4}$ J	0.5
	2.3		$P_{average} = \frac{ME_{lost}}{\Delta t} = \frac{1.95\times10^{-3} - 9.67\times10^{-4}}{0.52}$ Then, $P_{average} = 1.89 \times 10^{-3}$ W	0.5 0.5

Exercise 2 (5 pts) **Characteristics of electric components**

Part		Answer	Mark
1	1.1	Charging the capacitor	0.25
	1.2	$q = C u_C$ and $i = +\frac{dq}{dt}$, then $i = C\frac{du_C}{dt}$ $u_{AM} = u_{AB} + u_{BM}$, thus $E = u_C + R i = u_C + RC\frac{du_C}{dt}$ But, $\tau = RC$, hence $E = u_C + \tau\frac{du_C}{dt}$	0.75
	1.3	$u_C = 2(1 - e^{-1000t})$, then $\frac{du_C}{dt} = 2000\ e^{-1000t}$ Substituting in the above differential equation gives: $E = 2 - 2\ e^{-1000t} + \tau \times 2000\, e^{-1000t}$ $E = 2 + (-2 + 2000\,\tau)e^{-1000t}$, but $e^{-1000t} = 0$ is rejected Then, $-2 + 2000\,\tau = 0$, hence $\tau = 10^{-3}$ s	1
	1.4	$\tau = RC$, then $C = \frac{\tau}{R} = \frac{10^{-3}}{1000}$, so $C = 10^{-6}$ F = 1 μF	0.5
2	2.1	$u_{AB} = u_{AF} + u_{FN} + u_{NB}$, then $\frac{q}{C} = 0 + L\frac{di}{dt} + 0$ $i = -\frac{dq}{dt} = -q'$, then $i' = -\frac{d^2q}{dt^2} = -q''$ $\frac{q}{C} = -\ L q''$, then : $q'' + \frac{1}{LC} q = 0$	1
	2.2	The differential equation is of the form of : $q'' + \omega_0^2\, q = 0$, then $\omega_0 = \frac{1}{\sqrt{LC}}$ $T_0 = \frac{2\pi}{\omega_0}$, then $T_o = 2\pi\sqrt{LC}$	0.5
	2.3	Graphically, $T_E = 0.5\,\pi$ ms , then $T_0 = 2T_E = 2 \times 0.5\,\pi$, so $T_o = \pi$ ms	0.5
	2.4	$T_0^2 = 4\pi^2 L C$, then $(\pi \times 10^{-3})^2 = 4\pi^2 L (10^{-6})$, so L = 0.25 H	0.5

Exercise 3 (5 pts) **Self induction**

Part			Answer	Mark
1			**Statement 1 corresponds to circuit 2** **Statement 2 corresponds to circuit 1**	0.25 0.25
2	2.1		$u_{DA} = u_{DB} + u_{BA}$ $E = u_R + ri + L\frac{di}{dt}$ $u_{BD} = u_R = R\,i$, then $i = \frac{u_R}{R}$, hence $\frac{di}{dt} = \frac{1}{R}\frac{du_R}{dt}$ Then, $E = u_R + r\frac{u_R}{R} + \frac{L}{R}\frac{du_R}{dt}$ So: $\frac{L}{R}\frac{du_R}{dt} + \left(\frac{R+r}{R}\right)u_R = E$	1.25
	2.2		In the steady state: i = constant; then, $\frac{du_R}{dt} = 0$, and i is maximum , then $u_R = U_{Rmax}$ Substituting in the differential equation gives : $0 + \left(\frac{R+r}{R}\right)U_{R\,max} = E$, thus $U_{R\,max} = E\frac{R}{R+r}$	1
		1	$U_{R\,max} = 8$ V	0.25
		2	$U_{R\,max} = E\frac{R}{R+r}$, so $8 = 10\frac{8}{8+r}$, hence $r = 2\ \Omega$	0.75
		3	At $t = \tau$: $u_R = 63\ \%\ U_{Rmax} = 0.63 \times 8 = 5.04$ V Graphically : for $u_R = 0.63\ U_{Rmax} = 5.04$ V , $t = \tau = 50$ ms	0.25 0.5
		4	$\tau = \frac{L}{R+r}$, thus $L = \tau(R + r) = 0.05 \times (8 + 2)$, so $L = 0.5$ H	0.5

Exercise 4 (6 pts) **Stray bullets**

Part		Answer	Mark
1	1.1	$\vec{P}_o = m\overrightarrow{V_0} = 7 \times 10^{-3} \times 610\ \vec{j}$, then $\vec{P}_o = 4.27\ \vec{j}$ (kg.m/s) $\overrightarrow{P_1} = m\overrightarrow{V_1} = \vec{0}$, since $\overrightarrow{V_1} = \vec{0}$	**0.5** **0.25**
	1.2	$\Delta\vec{P} = \vec{P_1} - \vec{P_0} = \vec{0} - 4.27\ \vec{j}$, so $\Delta\vec{P} = -4.27\ \vec{j}$ (kg.m/s)	**0.5**
	1.3	$m\vec{g} + \vec{f} = \frac{\Delta\vec{P}}{\Delta t}$ Projecting the vectors along the y-axis gives : $-mg - f = \frac{\Delta P}{\Delta t}$ Then, $-7 \times 10^{-3} \times 10 - f = \frac{-4.27}{9.84}$, thus $f \cong 0.364$ N	**0.75**
	1.4	$ME_0 = KE_0 + PE_{g0} = \frac{1}{2} m V_o^2 + m g h_o = \frac{1}{2} \times (7 \times 10^{-3}) \times 610^2 + 0$ Then , $ME_o = 1302.35$ J	**0.5**
	1.5	$ME_1 = KE_1 + PE_{g1} = \frac{1}{2} m V_1^2 + mgh = 0 + 7 \times 10^{-3} \times 10\ h = 0.07\ h$ $\Delta ME = W_{\vec{f}}$, then $ME_1 - ME_0 = -f \times h$ $0.07\ h - 1302.35 = -0.364\ h$; therefore , $h \cong 3000$ m	**0.75**
	1.6	$\Delta ME = W_{\vec{f}} = -f\ h = -0.364 \times 3000 = -1092$ J $W_{th1} = \lvert\Delta ME\rvert = 1092$ J	**0.25**
2	2.1	$W_{th2} = \lvert W_{\vec{f_1}} \rvert$, and $W_{\vec{f}} = -f_1 \times BO$ BO = AO – AB = 3000 – 352 = 2648 m $W_{\vec{f}} = -0.07 \times 2648 = -185.36$J $\cong -185$ J , then $W_{th2} \cong 185$ J	**0.75**
	2.2	$W_{thermal} = 18 + 185 = 203$ J	**0.25**
3		For S : $V_{ground} = 44$ m/s < 61 m/s For S' : $V_{ground} = 90$ m/s > 61 m/s Therefore, S' is more dangerous than S	**0.5**