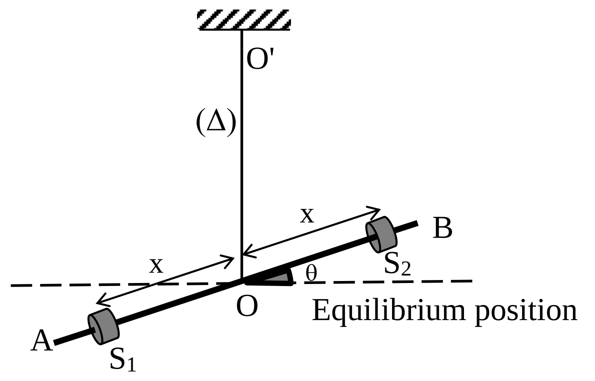
مسابقة في مادة الفيزياء	الاسم:
المدة: ساعتان ونصف	الرقم:

This exam is formed of four obligatory exercises in four pages.
The use of non-programmable calculator is recommended.

Exercise 1 (5 pts) Torsion pendulum

Consider a torsion pendulum (P) formed by:

- a uniform rod AB suspended from its center of mass O to a vertical torsion wire fixed from its upper end to a point O';
- two identical objects, (S_1) and (S_2), taken as particles of same mass m = 200 g. The two particles are fixed on the rod on opposite sides of O at the same adjustable distance « x » from it (Doc. 1).



Doc. 1

Neglect the mass of the torsion wire. The wire has a torsion constant C, and the rod AB has a moment of inertia I_0 about an axis (Δ) confounded with (OO').

The rod is rotated from its equilibrium position by an angle θ_m in the horizontal plane, and then it is released from rest.

The rod starts oscillating without friction in a horizontal plane about (Δ). At time t, the angular abscissa of the rod is θ and its angular velocity is $\theta' = \dfrac{d\theta}{dt}$.

The horizontal plane containing the rod is taken as a reference level for gravitational potential energy.

Given: $\pi^2 = 10$

1) Write the expression of the moment of inertia I of (P) about (Δ) in terms of I_0, m, and x.

2) Write the expression of the mechanical energy ME of the system [(P) , Earth] in terms of I, θ, C, and θ′.

3) Determine the differential equation that governs the variation of θ.

4) Deduce the expression of the proper (natural) period T_0 in terms of I and C.

5) Show that: $T_0^2 = \dfrac{4\pi^2 I_0}{C} + \dfrac{8\pi^2 m\, x^2}{C}$

6) We vary the distance x, and we measure the duration of 10 complete oscillations for each value of x. We record the measured values in the table of document 2.

x (cm)	10	15	20	25
Duration of 10 oscillations (s)	5.83	6.24	6.78	7.41
T_0 (s)				
T_0^2 (s^2)				
x^2 (m^2)				

Doc. 2

6.1) Copy and complete the table of document 2.

6.2) Draw on the graph paper the curve that represents T_0^2 as a function of x^2 using the following drawing scale: On the axis of abscissa: 1 cm ↔ 0.01 m^2
On the axis of ordinate: 1 cm ↔ 0.1 s^2

6.3) The shape of this curve can be considered in agreement with the expression of T_0^2 in part (5). Justify.

7) Deduce the values of I_0 and C.

Exercise 2 (5 pts)

Period of a simple pendulum

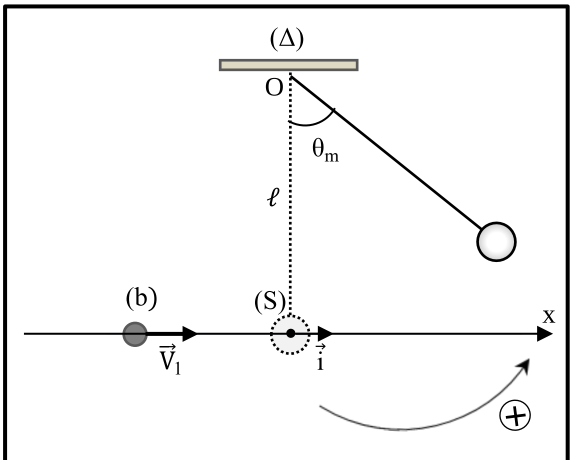
A simple pendulum is formed of a sphere (S), taken as a particle of mass $m_s = 2$ kg, suspended from a light inextensible cord of length $\ell = 1$ m.

A marble (b) of mass $m_b = 50$ g is launched with a velocity $\vec{V}_1 = 11\,\vec{i}$ (m/s) along a horizontal x-axis of unit vector $\vec{i}$, and it makes a head-on collision with (S) initially at rest.

Just after the collision, marble (b) recoils horizontally with a velocity $\vec{V}'_1 = -\,10.46\,\vec{i}$ (m/s) and (S) starts moving with a horizontal velocity of magnitude V_0.

The pendulum [Cord , (S)] oscillates without friction in a vertical plane about a horizontal axis (Δ) passing through the upper end O of the cord (Doc. 3).

The purpose of this exercise is to determine the oscillation period of the pendulum for different values of the marble's launch speed.



Doc. 3

Take:

- the horizontal plane passing through the lowest position of (S) as a reference level for gravitational potential energy;
- $\sin\theta \cong \theta$ in radians, for $\theta \leq 0.175$ rad;
- $g = 10$ m/s^2.

1) Collision between (S) and (b)

1.1) Prove that $V_0 = 0.537$ m/s by applying the principle of conservation of linear momentum to the system [(S) , (b)].

1.2) Show that this collision is elastic.

2) Maximum deflection of the pendulum

After the collision, the pendulum is deflected by a maximum angle θ_m. Show that $\theta_m = 0.17$ rad.

3) Oscillation of the pendulum

After the collision, the pendulum [Cord, (S)] oscillates in the vertical plane about (Δ). At an instant t, the angular abscissa of the pendulum is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

The differential equation the governs the variation of θ is: $\theta'' + \frac{g}{\ell}\sin\theta = 0$.

3.1) Deduce that the motion of the pendulum is simple harmonic.

3.2) Deduce the expression of the proper (natural) period T_0 of the oscillations in terms of ℓ and g.

3.3) Calculate the value of T_0.

4) The same experiment is repeated by lunching the marble horizontally with a velocity $\vec{V}_1 = V_1\,\vec{i}$, where $V_1 < 11$ m/s. Specify whether the value of the oscillation period of the pendulum increases, decreases or remains the same.

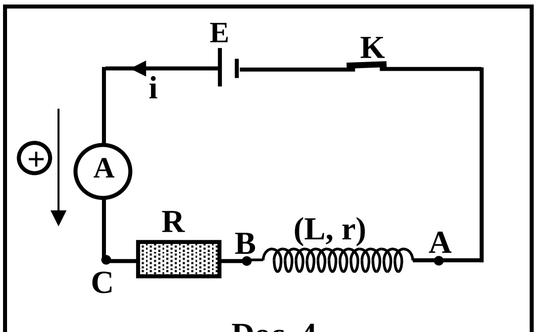
Exercise 3 (5 pts)

Characteristics of a coil

The circuit of document 4 consists of:

- an ideal battery of emf $E = 12$ V;
- an ohmic conductor of resistance $R = 15\ \Omega$;
- a coil of inductance L and resistance r;
- an ammeter (A) of negligible resistance;
- a switch K.

The purpose of this exercise is to determine the values of L and r.
At the instant $t_0 = 0$, switch K is closed and the current i in the circuit starts increasing gradually.
At the instant t_1 steady state is attained in the circuit, and ammeter (A) reads a current $I_1 = 0.5$ A.

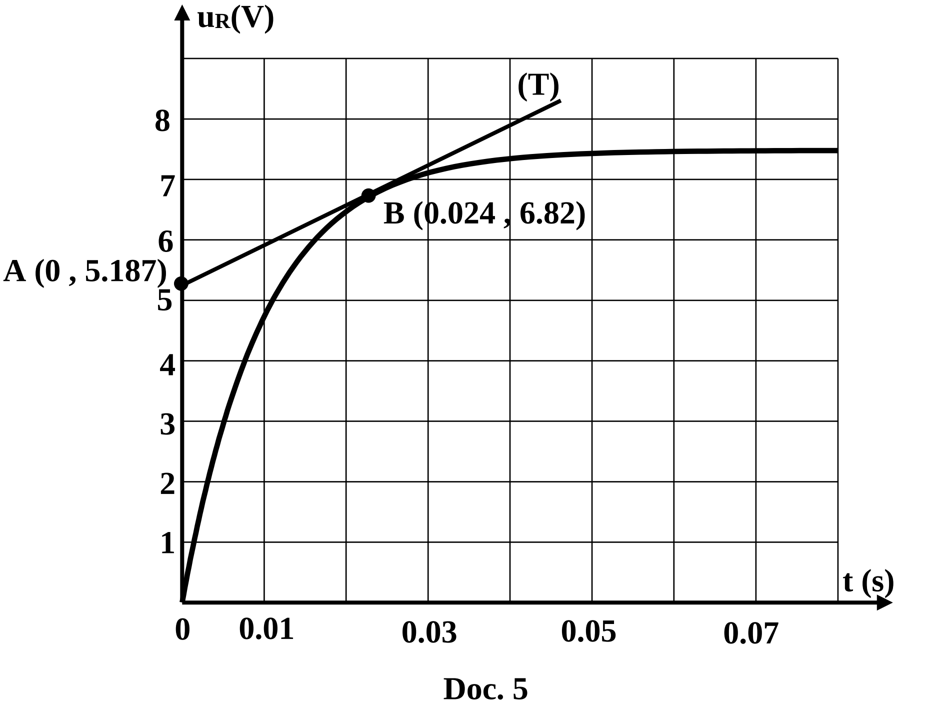


Doc. 4

1) The phenomenon of self-induction takes place in the coil between t_0 and t_1. Explain this phenomenon.
2) In steady state, the coil acts as a resistor of resistance r. Justify.
3) Show that the resistance of the coil is $r = 9\ \Omega$.
4) Show that the differential equation that governs the variation of the voltage $u_{CB} = u_R$ is:

$$\frac{RE}{L} = \left(\frac{r+R}{L}\right)u_R + \frac{du_R}{dt}$$

5) Verify that $u_R = \frac{RE}{r+R}\left(1 - e^{-\frac{t}{\tau}}\right)$ is a solution of this differential equation where $\tau = \frac{L}{r+R}$.
6) Deduce the expression of the instant t_1 in terms of L, r, and R.
7) The curve of document 5 represents u_R as a function of time.
(T) is the tangent to the curve u_R at point B (0.024s , 6.82V).
7.1) Calculate the slope of the tangent (T).
7.2) Use document 5 and the differential equation in order to deduce the value of L.



Doc. 5

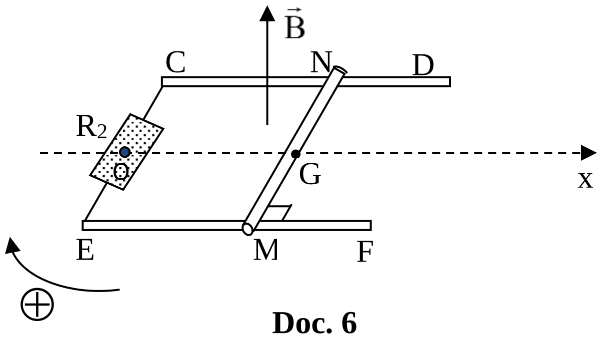
Exercise 4 (5 pts)

Electromagnetic induction

Two parallel conducting rails, CD and EF, of negligible resistance and separated by a distance $\ell = 15$ cm, are placed in a horizontal plane.

A rigid conducting rod MN, of length ℓ and perpendicular to the rails, may move without friction on the rails. The center of mass G of the rod moves along an x-axis which is parallel to the rails. The resistance of the rod is $R_1 = 0.5\ \Omega$. The ends C and E of the rails are connected to a resistor of resistance $R_2 = 0.5\ \Omega$.

The circuit formed by the two rails and the rod is placed in a vertical uniform magnetic field $\vec{B}$ perpendicular to the plane of the rails and of magnitude B = 0.5 T (Doc. 6).



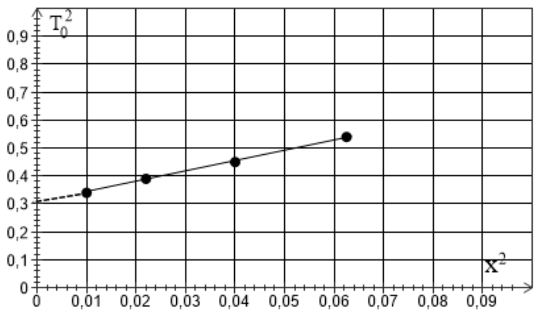
Doc. 6

At the instant $t_0 = 0$, G coincides with the origin O of the x-axis, and the rod is displaced at a constant velocity $\vec{V}$ in the positive x-direction.

At an instant t, the abscissa of G is $x = \overline{OG} = 2\,t$ (x in m and t in s).

1) The magnetic flux crossing the closed circuit CNME changes.
   - **1.1)** Indicate the reason behind the change in the magnetic flux crossing this circuit.
   - **1.2)** Explain this statement « The circuit CNME carries an electric current as long as the rod MN is moving ».
2) Show that the expression of the magnetic flux crossing the area CNME is $\phi = -0.15\,t$ (SI).
3) Determine the value of the electromotive force « e » induced in the rod.
4) Knowing that $u_{NM} = R_1 i - e$, show that the expression of the induced electric current in the circuit CNME is: $i = \dfrac{e}{R_1 + R_2}$
5) Deduce the value and the direction of i.
6) An electromagnetic force (Laplace's force) $\vec{F}$ is acting on the moving rod MN.
   - **6.1)** Indicate the direction of this force.
   - **6.2)** Calculate the magnitude F of $\vec{F}$.
7) We move the rod with a constant velocity having the same magnitude as that of $\vec{V}$ but of opposite direction. Indicate in this case the direction and the magnitude of the Laplace force acting on the rod.

مسابقة في مادة الفيزياء
المدة: ساعتان ونصف

<table>
<tr><th colspan="3">Exercise 1 (5 pts) — Torsion pendulum</th></tr>
<tr><th>Part</th><th>Answer</th><th>Mark</th></tr>
<tr><td>1</td><td>$I_{[P]/\Delta} = I_{rod/\Delta} + I_{S1/\Delta} + I_{S2/\Delta} = I_0 + mx^2 + mx^2 = I_0 + 2mx^2$</td><td>0.25</td></tr>
<tr><td>2</td><td>$ME = \frac{1}{2}I\theta'^2 + \frac{1}{2}C\theta^2$</td><td>0.5</td></tr>
<tr><td>3</td><td>ME= const , so $\frac{dE_m}{dt} = 0$, then
$I\theta'\theta'' + C\theta\theta' = 0$ then $\theta'' + \frac{C}{I}\theta = 0$</td><td>0.5</td></tr>
<tr><td>4</td><td>The differential equation is of the form : $\theta'' + \omega_0^2\theta = 0$, where $\omega_0 = \sqrt{\frac{C}{I}}$
So, $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{C}}$</td><td>0.25
0.25</td></tr>
<tr><td>5</td><td>$T_0 = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{I_0 + 2mx^2}{C}}$
$T_0^2 = \frac{4\pi^2 I_0}{C} + \frac{8\pi^2 m x^2}{C}$</td><td>0.25</td></tr>
<tr><td>6.1</td><td>
<table>
<tr><td>x (cm)</td><td>10</td><td>15</td><td>20</td><td>25</td></tr>
<tr><td>Duration of 10 oscillations</td><td>5.83</td><td>6.24</td><td>6.78</td><td>7.41</td></tr>
<tr><td>T_0 (s)</td><td>0.583</td><td>0.624</td><td>0.678</td><td>0.741</td></tr>
<tr><td>T_0^2 (s^2)</td><td>0.34</td><td>0.39</td><td>0.46</td><td>0.55</td></tr>
<tr><td>x^2 (m^2)</td><td>0.01</td><td>0.022</td><td>0.04</td><td>0.06</td></tr>
</table>
</td><td>0.75</td></tr>
<tr><td>6.2</td><td>
</td><td>0.75</td></tr>
<tr><td>6.3</td><td>The curve is a straight line not passing through the origin with a positive slope, so its equation is of the form : $T_0^2 = A x^2 + B$ (A and B are two positive constants).
Therefore, the curve is an agreement with the relation $T_0^2 = \frac{4\pi^2 I_0}{C} + \frac{8\pi^2 m x^2}{C}$</td><td>0.5</td></tr>
<tr><td>7</td><td>Slop of the curve $= \frac{8\pi^2 m}{C} = \frac{80 \times 0.02}{C} = \frac{0.55 - 0.34}{0.06 - 0.01} = 4.2$, then C $\cong$ 4 N.m/rad
By choosing a point on the curve : $I_0 \cong 0.03$ kg .m^2</td><td>0.5
0.5</td></tr>
</table>

Exercise 2 (5 pts)	Period of a simple pendulum	
Part	**Answer**	**Mark**
1.1	$\vec{P}_{before} = \vec{P}_{after}$; $m_b\vec{v}_1 = m_b\vec{v'}_1 + m_s\vec{v}_0$ $m_b\vec{v}_1 - m_b\vec{v'}_1 = m_s\vec{v}_0$; $m_b(\vec{v}_1 - \vec{v'}_1) = m_s\vec{v}_0$ $0.05\,(11\,\vec{i} + 10.46\,\vec{i}) = 2\,\vec{v}_0$; $\vec{v}_0 = 0.537\,\vec{i}$ (m/s)	**1**
1.2	$KE_{before} = \frac{1}{2}m_b v_1^2 = \frac{1}{2}\times 0.05\times 11^2 = 3.02$ J $KE_{after} = \frac{1}{2}m_b v_1'^2 + \frac{1}{2}m_S v_0^2 = \frac{1}{2}\times 0.05\times 10.46^2 + \frac{1}{2}\times 2\times 0.537^2 = 3.02$ J $KE_{before} = KE_{after}$, then the collision is elastic.	**1**
2	ME is constant, then : $\frac{1}{2}m_s v_0^2 = m_s g\,\ell\,(1 - \cos\theta_m)$ $\frac{1}{2}(0.537^2) = 10 \times 1 \times (1 - \cos\theta_m)$, then $\cos\theta_m = 0.986$, so $\theta_m = 0.17$ rad	**1**
3.1	$\theta m \leq 0.175$ rad , then $\sin\theta \approx \theta$, so $\theta'' + \frac{g}{\ell}\sin\theta = 0$. The differential equation is of the form : $\theta'' + \omega_0^2\theta = 0$ with $\omega_0 = \sqrt{\frac{g}{\ell}}$. Then the motion is simple harmonic.	**0.5**
3.2	$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{\ell}{g}}$	**0.5**
3.3	$T_0 = 2\pi\sqrt{\frac{1}{10}} = 1.99 \cong 2$ s	**0.5**
4	$V_1 < 11$ m/s, then $\theta m \leq 0.175$ rad, therefore the motion is still simple harmonic, hence $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$ which is independent of V_1. Then T_0 remains the same.	**0.5**

Exercise 3 (5 pts)	Characteristics of a coil	
Part	**Answer**	**Mark**
1	Between t_0 and t_1, the current increases, then the magnitude of the magnetic field produced inside the coil increases; hence, the coil is crossed by a variable self-flux. Therefore, the coil becomes the seat of induced emf.	**0.5**
2	$u_{BA} = ri + L\frac{di}{dt}$ In steady state, i = I_1 = constant , then $\frac{di}{dt} = 0$ Then, u_{BA} = ri ; therefore, the coil acts as a resistor.	**0.5**
3	$u_{CA} = u_{CB} + u_{BA}$, then E = Ri + ri +L$\frac{di}{dt}$ In steady state : E = RI_1 + rI_1 $I_1 = \frac{E}{R+r} = \frac{12}{15+r} = 0.5$, then $15+r = \frac{12}{0.5} = 24$, so r=9Ω	**0.5**
4	$E = Ri + +ri + L\frac{di}{dt}$, then $E = L\frac{di}{dt} + (R+r)i$ $u_R = Ri$; then $i = \frac{u_R}{R}$ and $\frac{di}{dt} = \frac{1}{R}\frac{du_R}{dt}$ So : $E = \frac{L}{R}\frac{du_R}{dt} + (R+r)\frac{u_R}{R}$ Therefore : $\frac{RE}{L} = \left(\frac{r+R}{L}\right)u_R + \frac{du_R}{dt}$	**1**
5	$u_R = \frac{RE}{r+R}(1 - e^{\frac{-t}{\tau}})$ $\frac{du_R}{dt} = \frac{RE}{r+R} \times \frac{1}{\tau} \times e^{\frac{-t}{\tau}} = \frac{RE}{r+R} \times \frac{r+R}{L} \times e^{\frac{-t}{\tau}} = \frac{RE}{L}e^{\frac{-t}{\tau}}$ Replace in the differential equation : $\frac{RE}{L} = \left(\frac{r+R}{L}\right)\left(\frac{RE}{r+R} - \frac{RE}{r+R}e^{\frac{-t}{\tau}}\right) + \frac{RE}{L}e^{\frac{-t}{\tau}}$, then $\frac{RE}{L} = \frac{RE}{L} - \frac{RE}{L}e^{\frac{-t}{\tau}} + \frac{RE}{L}e^{\frac{-t}{\tau}}$ Then, 0 = 0 So, this is a solution for the differential equation .	**0.75**
6	$t_1 = 5\,\tau = \frac{5L}{r+R}$	**0.25**
7.1	slope $= \frac{\Delta u_R}{\Delta t} = \frac{6.82 - 5.187}{0.024 - 0} = 68\ V/s$	**0.5**
7.2	$\frac{RE}{L} = \left(\frac{r+R}{L}\right)u_R + \frac{du_R}{dt}$ At t = 0.024 s , u_R=6.82V and $\frac{du_R}{dt}$ = slope = 68V/s Replace in the differential equation: $\frac{15\times12}{L} = \left(\frac{24}{L}\right)6.82 + 68$, *then* L= 0.24H = 240mH	**1**

Exercise 4 (5 pts)		Electromagnetic induction	
Part		**Answer**	**Mark**
1	**1.1**	During its motion, the area swept by the rod changes, then the circuit is crossed by a variable magnetic flux.	**0.5**
	1.2	During the motion of the rod, the magnetic flux changes, then the rod becomes the seat of emf and the closed circuit carries an induced current.	**0.5**
2		$\phi = B.S.\cos(\vec{B}, \vec{n}) = B.(\ell x).\cos(\pi) = 0{,}5\times 0{,}15 \times 2\,t =$ $-0.15\,t$.	**0.5**
3		$e = -\frac{d\phi}{dt} = 0.15$ V	**0.75**
4		$u_{NM} = R_1 i - e = u_{CE} = -R_2\, i$, then $e = (R_1 + R_2)\, i$, so $i = \frac{e}{R_1 + R_2}$ **<u>or</u>** : $u_{NM} + u_{CB} + u_{EC} + u_{CN} = 0$ $R_1\, i - e + 0 + R_2\, i + 0 = 0$, then $i = \frac{e}{R_1 + R_2}$	**0.5**
5		$i = \frac{e}{R_1 + R_2} = \frac{0.15}{1} = 0.15A$ i > 0 , then the circuit carries a current in the chosen positive sense (clockwise)	**0.5** **0.5**
6	**6.1**	direction : to the left	**0.25**
	6.2	$F = i\,B\,\ell\,\sin(\pi/2) = 0.15 \times 0.5 \times 0.15 \times 1 = 0.011$ N	**0.5**
7		Direction: to the right Value : F = 0.011 N	**0.25** **0.25**