مسابقة في مادة الفيزياء | الاسم:
المدة: ساعتان ونصف | الرقم:

<u>This exam is formed of four obligatory exercises in four pages.</u>
<u>The use of non-programmable calculator is recommended.</u>

Exercise 1 (5.5 pts)

Collision

Consider two simple pendulums (S_1) and (S_2):

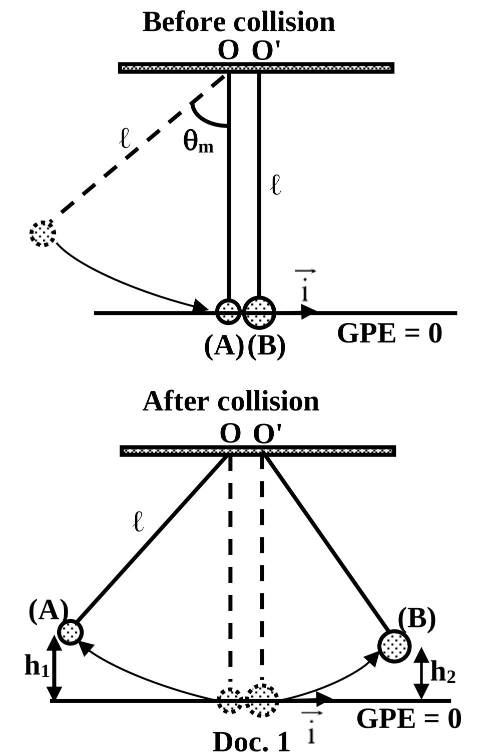
- pendulum (S_1) is formed of a sphere (A), taken as a particle of mass m_1, suspended to the lower extremity of a light inextensible string of length ℓ. The upper extremity of the string is fixed, at O, to a support;
- pendulum (S_2) is formed of a sphere (B), taken as a particle of mass $m_2 > m_1$, suspended to the lower extremity of a light inextensible string of same length ℓ. The upper extremity of the string is fixed, at O', to the same support of (S_1).

The two strings are vertical and the two particles touching each other (Doc. 1).

Neglect air resistance and friction around O and O'.

Take:

- the horizontal plane passing through the lowest positions of (A) and (B) as a reference level for gravitational potential energy;
- $g = 10 \text{ m/s}^2$.



Doc. 1

1) The pendulum (S_1) is shifted in the vertical plane by an angle θ_m from its stable equilibrium position, the string remains taut, and then (A) is released from rest. Determine, in terms of g, ℓ and θ_m, the expression of the speed v_1 of (A) when the pendulum (S_1) passes through the equilibrium position.

2) When (S_1) passes through the equilibrium position, (A) makes a head-on elastic collision with (B). Determine, in terms of m_1, m_2 and v_1, the expressions v'_1 and v'_2 of the algebraic values of the velocities $\vec{v'_1}$ and $\vec{v'_2}$ of (A) and (B) respectively just after this collision.

3) Specify the signs of v'_1 and v'_2.

4) P'_1 and P'_2 are respectively the algebraic values of the linear momentum of (A) and (B) just after this collision:

$P'_1 = \frac{-2}{75}$ kg.m/s and $P'_2 = \frac{1}{15}$ kg.m/s.

4.1) Determine the algebraic value of the linear momentum P_1 of (A) just before this collision.

4.2) Deduce the value of the angle θ_m if $\ell = 40$ cm and $m_1 = 20$ g.

5) The maximum heights attained by (A) and (B) after this collision are h_1 and h_2 respectively (Doc. 1).

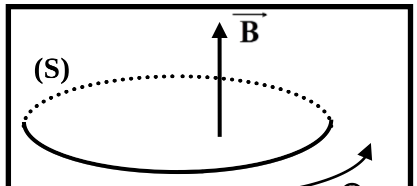
5.1) Determine the expression of h_1 in terms of v'_1 and g.

5.2) Write the expression of h_2 in terms of v'_2 and g.

5.3) Determine the ratio $\frac{m_1}{m_2}$ such that: $h_1 = h_2$.

Exercise 2 (5 pts)

Induction plate



Doc. 2

Consider a circular closed loop (S), of diameter d = 4 cm and of resistance R = 1 mΩ. The plane of the loop, placed horizontally, is perpendicular to a uniform magnetic field $\vec{B}$ (Doc. 2).

The magnetic field $\vec{B}$ is created by a current « i ».

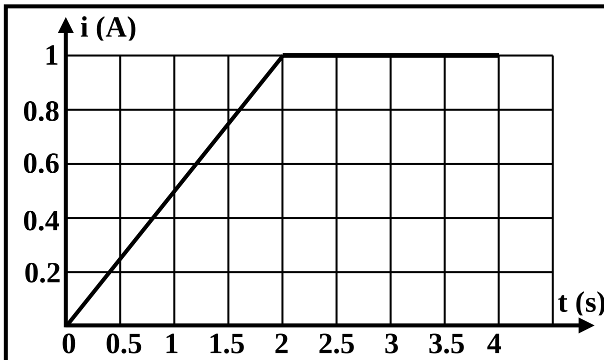
Document 3 shows « i » as a function of time.

The magnitude of the magnetic field is given by: B = 0.01 i (B in T and i in A)

1) Show that, taking into consideration the positive direction shown in document 2, the expression of the magnetic flux that crosses (S) is:

1.1) $\phi = 2\pi \times 10^{-6}\,t$ (ϕ in Wb and t in s) for $t \in [0\,;2\text{ s}]$;

1.2) $\phi = 4\pi \times 10^{-6}$ Wb for $t \in\,]2\text{ s}\,;4\text{ s}]$.



Doc. 3

2) Deduce the value of the induced electromotive force « e » that appears in (S) during each of the two intervals: [0; 2 s[and]2 s; 4 s].

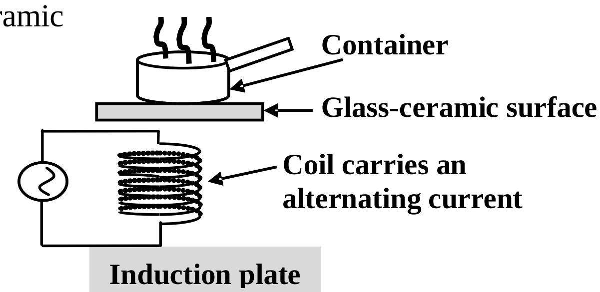
3) The electric energy produced in (S) is totally converted into thermal energy « E ».

3.1) Specify the time interval during which there is release of thermal energy in (S).

3.2) Determine, during this interval, the value of the released thermal energy « E », knowing that the induced current in (S) is given by: $i_1 = \dfrac{e}{R}$.

4) The induction plates are one of the applications for the production of thermal energy based on the principle of electromagnetic induction (Doc. 4).

In this type of plates, a coil is placed under a glass ceramic surface. When the coil carries a variable alternating current of an adjustable frequency f between 50 Hz and 50 kHz, it generates a variable magnetic field which, in turn, induces electric currents in the metallic bottom of the container, which produces thermal energy (heat) by Joule's effect.



Doc. 4

4.1) Referring to document 4, indicate the part that acts as the inducing source and the one that acts as the induced circuit.

4.2) The bottom of the container is modeled by a circular loop similar to (S). Explain the existence of an induced current in the bottom of the container.

4.3) The average total power produced in the bottom of the container is given by :

$$P = k\,(2\pi f)^2 \text{ (k is a positive constant).}$$

Determine the factor by which the power P will be multiplied if the frequency f becomes 100 times greater.

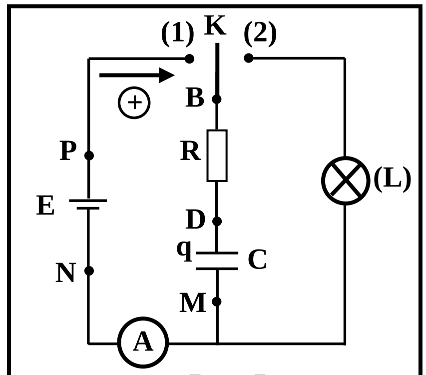
Exercise 3 (5 pts)

Duration of the brightness of a lamp

The aim of this exercise is to study the duration of brightness of a lamp on the ceiling of a car.

For this purpose, we set up the circuit of document 5 that includes:

- an ideal battery of electromotive force E = 12 V;
- a resistor of resistance R;
- a capacitor, initially uncharged, of capacitance C;
- an ammeter (A) of negligible resistance;
- a lamp (L) of negligible resistance;
- a double switch K.



Doc. 5

1) Charging the capacitor

At instant $t_0 = 0$, K is in position (1) and the charging process of the capacitor starts.

At an instant t, the plate D of the capacitor carries a charge q and the circuit carries a current i.

The differential equation that describes the variation of the current i is: $R\frac{di}{dt} + \frac{1}{C}i = 0$.

1.1) Verify that $i = \frac{E}{R} e^{\frac{-t}{RC}}$ is a solution of the differential equation.

1.2) Deduce the expression of i at $t_0 = 0$.

1.3) Calculate the value of R, knowing that at $t_0 = 0$, the ammeter indicates 1.2 mA.

1.4) Apply the law of addition of voltages to prove that $q = EC - EC\,e^{\frac{-t}{RC}}$.

1.5) When the capacitor is fully charged, the charge of plate D is $Q = 12\times10^{-4}$ coulomb. Show that $C = 100\ \mu F$.

1.6) Calculate the value of the time constant τ of the circuit during charging the capacitor, knowing that $\tau = RC$.

2) Discharging the capacitor

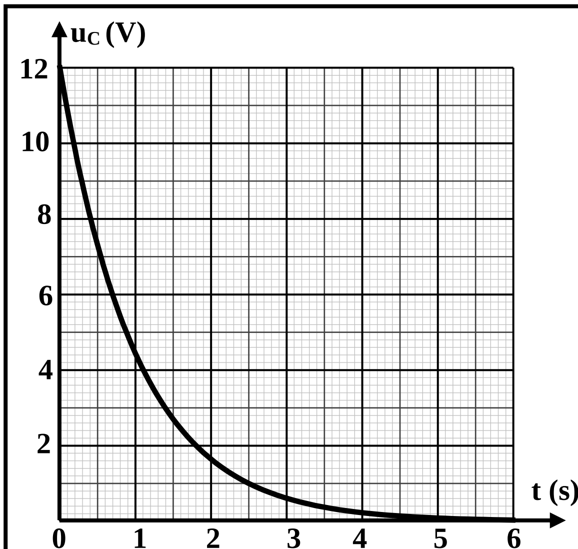
The circuit in document 5 represents a model circuit, used to turn on a lamp in the ceiling of a car.

Initially the switch is in position (1) and the capacitor is fully charged, when the door of the car is opened and then closed, the switch is turned to position (2).

Thus the capacitor discharges through the resistor and the lamp.

During discharging the brightness of the lamp decreases gradually.

Document 6, shows the voltage $u_{DM} = u_C = E\ e^{\frac{-t}{\tau'}}$ as a function of time.



Doc. 6

2.1) Using document 6, determine the value of the time constant τ' of the circuit during the discharging of the capacitor.

2.2) $\tau = \tau'$. Why?

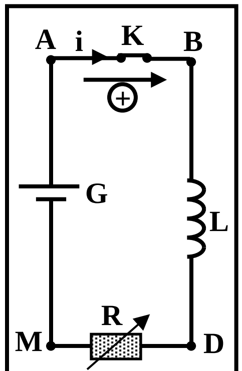
2.3) The lamp glows as long as $u_C \geq 1$ V.
Referring to document 6, indicate how long the lamp glows during the dicharging of the capacitor.

2.4) Specify how the value of C should be varied in order to increase this duration.

Exercise 4 (4.5 pts)

Inductance of a coil

The aim of this exercise is to determine the inductance L of a coil. For this purpose, we set up the series circuit represented in document 7.



Doc. 7

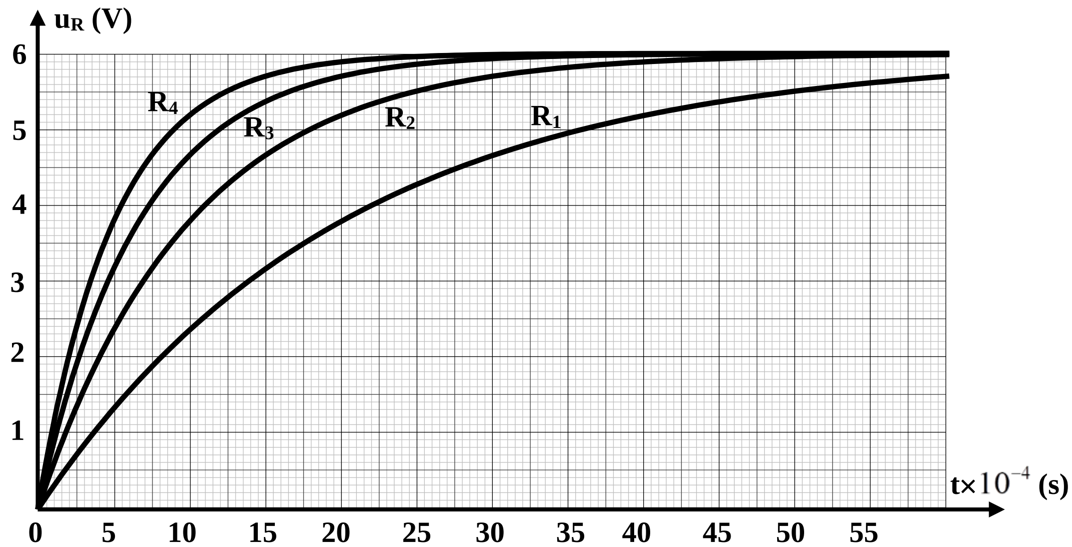
This series circuit is composed of: an ideal battery (G) of electromotive force E = 6 V, a switch K, a resistor of adjustable resistance R, and a coil of inductance L and of negligible resistance.

Switch K is closed at $t_0 = 0$. At an instant t, the circuit carries a current i.

1) Show that the first order differential equation that describes the variation of the voltage $u_R = u_{DM}$ across the resistor is given by: $E = \frac{L}{R}\frac{du_R}{dt} + u_R$.

2) The solution of this differential equation has the form: $u_R = a + b\ e^{\frac{-t}{\tau}}$ where a, b and τ are constants. Determine the expressions of a, b and τ in terms of R, E and L.

3) Document 8 shows u_R as a function of time for four different values of R:
$R_1 = 50\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 150\ \Omega$ and $R_4 = 200\ \Omega$



Doc. 8

3.1) Using document 8, determine, for each value of R, the value of the time constant τ of the R-L circuit.

Value of R	$R_1 = 50\ \Omega$	$R_2 = 100\ \Omega$	$R_3 = 150\ \Omega$	$R_4 = 200\ \Omega$
Time constant τ (s)	$\tau_1 =$	$\tau_2 =$	$\tau_3 =$	$\tau_4 =$

3.2) Verify that $R_1\tau_1 = R_2\tau_2 = R_3\tau_3 = R_4\tau_4$.

3.3) Deduce the value of L.

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أسس التصحيح

Exercise 1 (5.5 pts)	**Collision**	
Part	**Answer**	**Note**
1	No friction (or the work done by the nonconservative force is zero), the mechanical energy of the system [(S_1), Earth] is conserved. $(ME)_{\theta = \theta m} = (ME)_{\theta = 0}$, So, $(KE + GPE)_{\theta = \theta m} = (KE + GPE)_{\theta = 0}$ then $(GPE)_{\theta = \theta m} = (KE)_{\theta = 0}$ $m_1 g\ell(1 - \cos\theta) = \frac{1}{2} m_1 v_1^2$, this implies $v_1 = \sqrt{2g\ell(1-\cos\theta_m)}$	**0.75**
2	During collision, linear momentum is conserved. $\vec{P}_{\text{just before collision}} = \vec{P}_{\text{just after collision}}$; $m_1 \vec{v}_1 = m_1 \vec{v'}_1 + m_2 \vec{v'}_2$ Collinear velocities, Project along $\vec{i}$ (algebraic values): $m_1 v_1 = m_1 v'_1 + m_2 v'_2$, so $m_1 (v_1 - v'_1) = m_2 v'_2$ ….. Eq. (1) Elastic collision, kinetic energy is conserved: $KE_{\text{before collision}} = KE_{\text{after collision}}$ $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$, so $m_1 (v_1^2 - v_1'^2) = m_2 v_2'^2$ ….. Eq. (2) Divide Eq. (2) by Eq. (1): $\frac{m_1(v_1^2 - v_1'^2)}{m_1(v_1 - v'_1)} = \frac{m_2 v_2'^2}{m_2 v'_2}$, we get: $v'_2 = v_1 + v'_1$ ….. Eq. (3) Solve Eq. (1) and Eq. (3) to find v'_1 and v'_2: $v'_1 = \frac{(m_1 - m_2)v_1}{m_1 + m_2}$ and $v'_2 = \frac{2m_1 v_1}{m_1 + m_2}$	**1.25**
3	$v'_1 < 0$ since $m_1 < m_2$ and v_1 is positive; so (A) moves in the negative sense (rebounds) $v'_2 > 0$ since m_1 and v_1 are positive; so, (B) moves in the positive direction	**0.25** **0.25**
4.1	During collision, linear momentum is conserved $\vec{P}_{\text{just before collision}} = \vec{P}_{\text{just after collision}}$ Algebraic values : $P_{\text{just before collision}} = P'_1 + P'_2 = 0.04$ kg.m/s $= \frac{1}{25}$ kg. m/s	**0.5**
4.2	$P_{\text{just before collision}} = m_1 v_1$, so $0.04 = 0.02 \times v_1$ this implies = 2 m/s But $v_1 = \sqrt{2g\ell(1-\cos\theta_m)}$, so $2 = \sqrt{2\times10\times0.4(1-\cos\theta_m)}$ this implies $\theta_m = 60^o = 1.04$ rad	**0.25** **0.5**
5.1	System [(S_1), Earth] ; $ME_{h = 0} = ME_{h = h1}$ $(KE)_{h = 0} = (GPE)_{h = h1}$; then $\frac{1}{2} m_1 v_1'^2 = m_1 g h_1$ this implies $h_1 = \frac{v_1'^2}{2g}$	**0.5**
5.2	$h_2 = \frac{v_2'^2}{2g}$	**0.25**
5.3	$h_1 = h_2$ then $v_1'^2 = v_2'^2$. But after collision, (A) and (B) moves in opposite directions. Hence $\frac{2m_1 v_1}{m_1 + m_2} = -\frac{(m_1 - m_2)v_1}{m_1 + m_2}$. This implies, $2 m_1 v_1 = -m_1 v_1 + m_2 v_1$, So, $3 m_1 v_1 = m_2 v_1$, thus: $\frac{m_1}{m_2} = \frac{1}{3}$	**1**

Exercise 2 (5 points) Induction plate		
Part	**Answer**	**Note**
1.1	For $t \in [0\ ;\ 2\ s]$: $i = 0.5\ t$ $\phi = B\times S \times \cos(\vec{B}, \vec{n}) = 0.01\ i \times\pi \times r^2\times \cos(0^o) = 0.01\times 0.5\ t \times\pi \times 0.02^2 = 2\pi \times10^{-6}\ t$ Wb	**0.25** **0.75**
1.2	For $t \in]2\ s\ ;\ 4\ s]$: $i = 1$ A $\phi = B\times S\times \cos(\vec{B}, \vec{n}) = 0.01\ i \times \pi \times r^2\times \cos(0^o) = 0.01\times 1 \times \pi \times 0.02^2 = 4\pi \times 10^{-6}$ Wb	**0.25** **0.5**
2	$e = -\dfrac{d\phi}{dt}$ For $t \in [0\ ;\ 2s[$: $e = -6.3 \times10^{-6}$ V $= -2\pi \times10^{-6}$ V For $t \in]2s\ ;\ 4s]$: $e = 0$ V	**0.25** **0.25** **0.25**
3.1	There is release of thermal energy when (S) carries an induced current, that is when we have induced e.m.f , hence thermal energy is released in the interval [0 ; 2s[	**0.25**
3.2	$i_1 = \dfrac{e}{R} = \dfrac{-6.3\times10^{-6}}{0.001} = -6.3\times10^{-3}$ A $= -2\pi \times10^{-3}$ A $E = R\ i_1^2 \times t = 0.001 \times (6.3\times10^{-3})^2 \times 2 = 7.9\times10^{-8}$ J	**0.25** **0.5**
4.1	Source of induction: the coil Induction circuit : Bottom of the container	**0.25** **0.25**
4.2	When the coil carries a variable alternating current, it creates a variable magnetic field, so we have a variable flux through the bottom of the container, which induces electric current in the bottom of the metal of the container.	**0.5**
4.3	$f' = 100\ f$ $P = k\ (2\pi\ f)^2$; $P' = k\ (200\ \pi\ f)^2 = 10^4\ k\ (2\pi\ f)^2 = 10^4\ P$	**0.5**

Exercise 3 (5 points)	Duration of the brightness of a lamp	
Part	**Answer**	**Note**
1.1	$R\frac{di}{dt}+\frac{1}{C}i=0$; $\frac{di}{dt}=-\frac{E}{R}\frac{1}{RC}e^{\frac{-t}{RC}}$, replace i and $\frac{di}{dt}$ in the differential equation $-R\frac{E}{R}\frac{1}{RC}e^{\frac{-t}{RC}}+\frac{1}{C}\frac{E}{R}e^{\frac{-t}{RC}}=0$ thus $-\frac{E}{RC}e^{\frac{-t}{RC}}+\frac{1}{C}\frac{E}{R}e^{\frac{-t}{RC}}=0$; $0=0$ (verified)	**1**
1.2	At $t_0=0$; $i_0=\frac{E}{R}$	**0.25**
1.3	$1.2\times10^{-3}=\frac{12}{R}$ so $R = 10\,000\ \Omega = 10\ k\Omega$	**0.25**
1.4	$E = u_R + u_C$; $E = Ri+\frac{q}{C}$; $E = E\,e^{\frac{-t}{RC}}+\frac{q}{C}$; $E-E\,e^{\frac{-t}{RC}}=\frac{q}{C}$; thus, $q = EC - EC\,e^{\frac{-t}{RC}}$	**1**
1.5	$Q = EC = 12\times10^{-4}$ C so $C = 1\times10^{-4}$ F = 100 μF.	**0.25**
1.6	$\tau = RC = 10\,000\times1\times10^{-4} = 1$ s	**0.25**
2.1	At $t=\tau'$, we have $u_C = 0.37\times12 = 4.44$ V which corresponds for $\tau' = 1$s	**0.75**
2.2	Since $\tau = RC$, C is the same and R is the same; while the lamp has negligible resistance, Then, $\tau = \tau'$.	**0.5**
2.3	$U_C = 1$ V for t = 2.5 s	**0.25**
2.4	To increase t, C must be increased because when C increases, τ increases. The capacitor takes more time to reach the voltage of 1 V.	**0.5**

Exercise 4 (4.5 pts)	**Inductance of a coil**	
Part	**Answer**	**Note**
1	$u_{AM} = u_{AB} + u_{BD} + u_{DM}$ $E = L\frac{di}{dt} + u_R$; $u_R = Ri$ and $\frac{di}{dt} = \frac{1}{R}\frac{du_R}{dt}$ so $E = \frac{L}{R}\frac{du_R}{dt} + u_R$	**0.75**
2	$u_R = a + b\,e^{\frac{-t}{\tau}}$; $\frac{du_R}{dt} = -\frac{b}{\tau}\,e^{\frac{-t}{\tau}}$ Replace in the differential equation : $E = \frac{-Lb}{R\tau}\,e^{\frac{-t}{\tau}} + a + b\,e^{\frac{-t}{\tau}}$ $b\,e^{\frac{-t}{\tau}}\left[\frac{-L}{R\tau} + 1\right] + a = E$ By comparison: $a = E$ and $\tau = \frac{L}{R}$ At $t = 0$, $i = 0$, so $u_R = 0$ and $a = -b$ thus $b = -E$	**1**
3.1	Graphical method to determine the time constant τ. At $t = \tau$ we have $u_R = 0.63\times6 = 3.78 \cong 3.8$ V ; From the graph for each value of R we get τ : <table><tr><td>R</td><td>$R_1 = 50\ \Omega$</td><td>$R_2 = 100\ \Omega$</td><td>$R_3 = 150\ \Omega$</td><td>$R_4 = 200\ \Omega$</td></tr><tr><td>τ (s)</td><td>$\tau_1 = 20\times10^{-4} = 0.002$ s</td><td>$\tau_2 = 0.001$ s</td><td>$\tau_3 = 0.0006$ s</td><td>$\tau_4 = 0.0005$ s</td></tr></table>	**0.5** **1**
3.2	Since, $R_1\tau_1 = 0.1$; $R_2\tau_2 = 0.1$; $R_3\tau_3 = 0.09 \cong 0.1$; and $R_4\tau_4 = 0.1$ (SI unit) Thus, $R_1\tau_1 = R_2\tau_2 = R_3\tau_3 = R_4\tau_4 \cong 0.1$ (S.I.)	**0.5**
3.3	Since $\tau = \frac{L}{R}$ so $R\tau = L$ this implies $L = 0.002\times50 = 0.1$ H	**0.75**