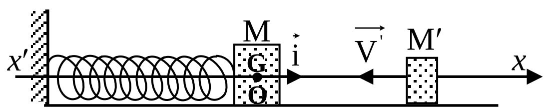
This exam is formed of three exercises in three pages.
The Use of non-programmable calculators is allowed.

First exercise (7 points)

Mechanical oscillator

A spring of un-jointed loops, of stiffness constant k = 10 N/m and of horizontal axis, is fixed from one extremity to a fixed obstacle; the other extremity is attached to a puck M of mass m = 100 g. The center of inertia G of M can slide, without friction, along a horizontal axis x'x of origin O and unit vector $\vec{i}$. The horizontal plane passing through G is taken as a gravitational potential energy reference.



At the instant $t_0 = 0$, the puck M, initially at rest at O, is hit with another puck M' of mass $m' = \frac{m}{2}$ moving initially with a velocity $\vec{V'} = -V'\vec{i}$ ($V' > 0$). After collision, the puck M' rebounds on M with a velocity $\vec{V'_1}$ and the puck M moves with a velocity $\vec{V_0} = V_0\vec{i}$, and performs oscillations with a constant amplitude $X_m = 10$ cm.

1) Give the sign of V_0.

2) Let x and v be respectively the algebraic values of the abscissa and the velocity of G at an instant t after the collision.

- **a)** Write, in terms of x, m, k and v, the expression of the mechanical energy of the system (M, spring, Earth) at the instant t.
- **b)** Derive the differential equation of second order in x that describes the motion of M.
- **c)** The solution of this differential equation is of the form $x = A\sin(\omega_0 t + \varphi)$. Determine the values of the positive constants A, ω_0 and φ.
- **d)** Deduce that the magnitude of the velocity $\vec{V_0}$ of M just after the collision is 1 m/s.

3) Knowing that the collision between M' and M is supposed to be perfectly elastic, determine:

- **a)** the value V' of the velocity of M' before collision;
- **b)** the velocity $\vec{V'_1}$ of M' just after the collision.

Second exercise (7 points)

Determination of the capacitance of a capacitor

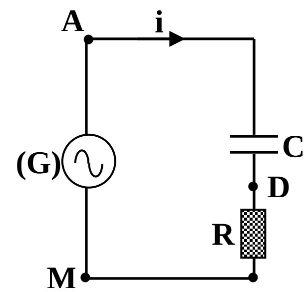


Figure 1

In order to determine the capacitance C of a capacitor, we connect it in series with a resistor of resistance $R = 10\sqrt{2}\ \Omega$ across the terminals of a low frequency generator (G) delivering across its terminals an alternating sinusoidal voltage $u_G = U_m \cos\omega t$.
The circuit thus constructed carries an alternating sinusoidal current i (Fig1).
Take $\sqrt{2} = 1.4$ and $0.32\pi = 1$.

1) Redraw the circuit of figure (1) and show the connections of the oscilloscope in order to display the voltages $u_G = u_{AM}$ across the generator and $u_R = u_{DM}$ across the resistor.
2) Which of the two voltages, u_G or u_R, represents the image of the current i ? Justify your answer.
3) In figure 2, the waveform (1) represents the variation of the voltage u_G with time.
   a) Specify, with justification, which of the voltages u_G or u_R, leads the other.
   b) Determine the phase difference between the voltages u_G and u_R.
4) Using the waveforms of figure 2, determine the angular frequency ω, the maximum value U_m of the voltage u_G and the maximum value I_m of the current i.
   Horizontal sensitivity: 5 ms/div.
   Vertical sensitivity on both channels: 1 V/div.
5) a) Write down the expression of the current i as a function of time t.
   b) Deduce the expression of the voltage $u_C = u_{AD}$ across the terminals of the capacitor as a function of C and t.
6) By applying the law of addition of voltages and giving the time t a particular value, determine the value of C.

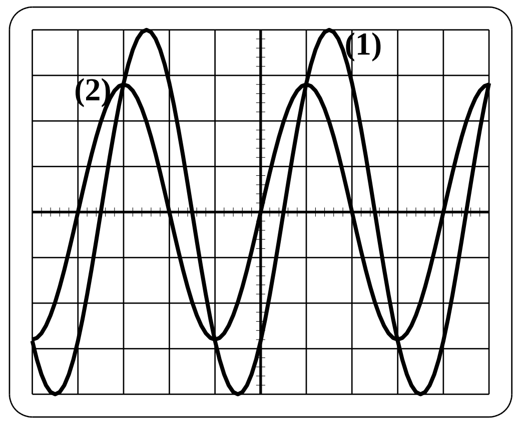
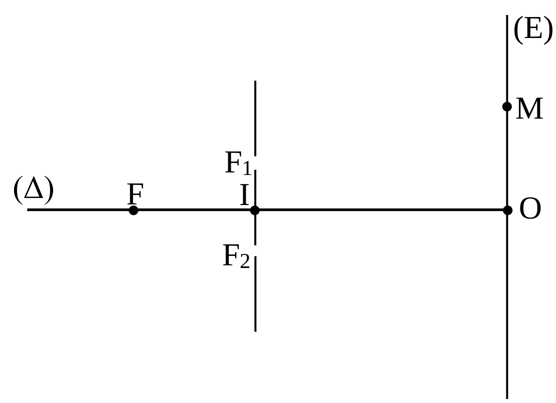


Figure 2

Third exercise (6 points)

Interference of light

Consider Young's experiment set-up that is formed of two very thin parallel slits F_1 and F_2, separated by a distance a = 1 mm , and a screen of observation (E) placed parallel to the plane of the slits at a distance D = 2 m from the mid point I of F_1F_2 and a thin slit F, equidistant from F_1 and F_2, situated on the straight line (Δ) whose intersection with (E) is the point O.
The object of this exercise is to study the interference pattern observed on the screen (E) in different situations.



A – First situation

The slit F is illuminated with a monochromatic light of wavelength $\lambda = 0.64\ \mu m$ in air.

1) Describe the interference pattern observed on (E).
2) Consider a point M on the screen at a distance d_1 from F_1 and d_2 from F_2.
   Specify the nature of the fringe thus formed at point M in each of the following cases:

a) $d_2 - d_1 = 0$;
b) $d_2 - d_1 = 1.28\ \mu m$;
c) $d_2 - d_1 = 0.96\ \mu m$.

3) F is moved along (Δ). We observe that the interference fringes remain in their positions. Explain why.

4) F is moved perpendicularly to (Δ) to the side of F_2. We observe that the central fringe is displaced. In which direction and why?

B – Second situation

Now the slit F is illuminated with white light.

1) We observe at point O a white fringe. Justify.

2) Specify the color of the bright fringe that is the nearest to the central fringe.

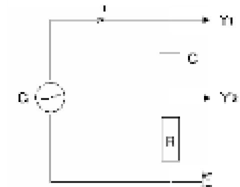
C – Third situation

Consider two lamps (L_1) and (L_2) emitting radiations of same wavelength, we illuminate F_1 by (L_1) and F_2 by (L_2), we observe that the system of interference fringes does not appear on the screen (E). Why?

First exercise (7 points)

Part of the Q	Answer	Mark
1	$V_0 < 0$.	0.25
2.a	Mechanical energy: ME = PE + KE = $\frac{1}{2}k\cdot x^2 + \frac{1}{2}m\cdot V^2$	0.50
2.b	Without friction ⇔ Conservation of mechanical energy $\Leftrightarrow$ ME = $\frac{1}{2}k\cdot x^2 + \frac{1}{2}m\cdot V^2$ = constant. By deriving with respect to time: $\frac{dE_m}{dt} = kx\dot{x} + mV\dot{V} = 0$; $\Leftrightarrow \ddot{x} + \frac{k}{m}x = 0$.	1.00
2.c	$x = A\sin(\omega_0 t + \varphi)$; $\dot{x} = A\omega_0\cos(\omega_0 t + \varphi)$ and $\ddot{x} = -A\omega_0^2\sin(\omega_0 t + \varphi)$ By replacing in the differential equation: $A\omega_0^2\sin(\omega_0 t + \varphi) + \frac{k}{m}A\sin(\omega_0 t + \varphi) = 0 \Leftrightarrow \omega_0^2 = \frac{k}{m} = \frac{10}{0.1} = 100$, $\omega_0 = 10$ rd/s. For $t_0 = 0$, $x = A\sin(\varphi) = 0$, then $\varphi = 0$ or π and $v = A\omega_0\cos(\varphi) = V_0 < 0$; as $A > 0$, then $\cos\varphi < 0 \Rightarrow \varphi = \pi$ rad and A = +10 cm.	1.50
2.d	$v = \dot{x} = -\omega_0 A\cos(\omega_0 t)$; at $t_0 = 0$, $v = V_0 = -\omega_0 x_m = -1$ m/s.	0.75
3	Conservation of linear momentum: $\Leftrightarrow \vec{P}_i = \vec{P}_f \Leftrightarrow m'\vec{V}' = m'\vec{V}'_1 + m\vec{V}_0$ In algebraic values: $V' = V'_1 + 2V_0$. (I) Elastic collision ⇔ Conservation of KE: $\Leftrightarrow \frac{1}{2}m'V'^2 = \frac{1}{2}m'V'^2_1 + \frac{1}{2}mV_0^2$ $\Leftrightarrow m'(V'^2 - V'^2_1) = mV_0^2$ (II) $\Leftrightarrow \frac{(II)}{(I)} \Leftrightarrow V' + V'_1 = V_0$ Substituting in(I) we obtain: $V' = \frac{3}{2}V_0 = 1.5\,V_0 = -1.5$ m/s.	2.00
4	$V'_1 = V_0 - V' = -1 - (-1,5) = 0.5$ m/s $\vec{V'_1} = 0.,5\,\vec{i}$	1.00

Second exercise (7 points)

Part of the Q	Answer	Mark
1	 Fig1	0.5
2	$u_R = Ri = ct\ i \Rightarrow u_R$ is the image of i.	0.50
3.a	u_R leads u_G , because in this circuit the current always leads the voltage across the generator.(u_R attains the maximum before).	0.5
3.b	$T \rightarrow 2\pi \rightarrow 4$ div. $\varphi \rightarrow 0.5$div=> $\varphi = \frac{\pi}{4}$ rad.	0.75
4	$T = 4\text{div} \times 5$ ms/div = 20 ms $\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{0.02} = 100\pi$ rad/s. $U_m = 4$ div $\times 1$ V/div = 4 V. $(U_R)_m = 2.8$ div $\times 1$ V/div = 2.8 V = $2\sqrt{2}$ V = R I_m $\Rightarrow I_m = \frac{2\sqrt{2}}{10\sqrt{2}} = 0.2$ A.	2
5 a)	$i = I_m \cos(\omega t + \frac{\pi}{4}) = 0.2 \cos(\omega t + \frac{\pi}{4})$	0.25
5 b)	$i = \frac{dq}{dt} = C\frac{du_C}{dt} \Rightarrow u_C = \frac{1}{C}$ primitive of $i = \frac{0.2}{100\pi C}\sin(\omega t + \frac{\pi}{4})$	1
6	$u_G = u_C + u_R$; $u_R = 2\sqrt{2}\cos(\omega t + \frac{\pi}{4})$ $4\cos\omega t = \frac{0.2}{100\pi C}\sin(\omega t + \frac{\pi}{4}) + 2\sqrt{2}\cos(\omega t + \frac{\pi}{4})$. For t = 0, we have : $4 = \frac{0.2}{100\pi C} \times \frac{\sqrt{2}}{2} + 2\sqrt{2}\,\frac{\sqrt{2}}{2} \Rightarrow$ $C = 224 \times 10^{-6}$F = 224 μF.	1.50

Third exercise (6 points)

Part of the Q	Answer	Mark
A.1	- Fringes are parallel to the slits - Fringes are alternately bright and dark - Fringes are equidistant	0.75
A.2.a	d_2-d_1 = 0 = kλ with k = 0 ; M is a bright central fringe.	0. 5
A.2.b	d_2-d_1 = 1.28 μm = k λ with k = 2 ;M is a bright fringe of order 2.	0.75
A.2.c	d_2-d_1 = 0.96 μm = (2k + 1) $\lambda / 2$ with k = 1 ; M is a dark fringe of order 1	0.75
A.3	FF_1 remains equal to FF_2, the optical path difference $\delta = \frac{ax}{D}$ does not vary thus the interfringe i does not vary.	0.75
A.4	$FF_1 > FF_2$; the optical path FF_1 M increases. To locate the central bright fringe O', we must have FF_1O' = FF_2O', the optical path F_2O' must increase $\Rightarrow$the central fringe is displaced to the side of F_1.	1
B.1	We see at O a white fringe since all the bright fringes corresponding to different colors superpose at O.	0.5
B.2	$x = k\frac{\lambda D}{a}$ **;** for k =1 **,** x is the smallest value corresponding to the smallest wavelength $\Rightarrow$ we observe a violet bright fringe.	0.75
C	No, since the two sources are not coherent.	0.25