| دورةٌ سنـة 2013 العادية | امتحانات الثشهادة الثلانويـة العامة الفرع : علوم الحياة | وزارة التربيةّ والتتعليم العالثي المديرية العامة للتربية دائرة الامتحانـات |
| :---: | :---: | :---: |
| الرقم: | مسابقة في مادة الفيزياء المدة ساعتان | الجمعة 28 حزيران 2013 |

## This exam is formed of three obligatory exercises in 3 pages numbered from 1 to 3 <br> The use of non-programmable calculator is recommended

## First exercise: (7 points)

## Collisions and mechanical oscillator

## A - Collision

A pendulum is formed of a massless and inextensible string of length $\ell=1.8 \mathrm{~m}$, having one of its ends C fixed to a support while the other end carries a particle $\left(\mathrm{P}_{1}\right)$ of mass $\mathrm{m}_{1}=200 \mathrm{~g}$.
The pendulum is stretched horizontally. The particle $\left(\mathrm{P}_{1}\right)$ at $\mathrm{A}_{0}$ is then launched vertically downward with a velocity $\vec{V}_{i}$ of magnitude $V_{i}=8 \mathrm{~m} / \mathrm{s}$.
At the lowest position $\mathrm{A},\left(\mathrm{P}_{1}\right)$ enters in a head-on perfectly elastic collision with another particle $\left(\mathrm{P}_{2}\right)$ of mass $m_{2}=300 \mathrm{~g}$ initially at rest. Neglect all frictional forces.


Take:

- the horizontal plane passing through A as a gravitational potential energy reference;
- $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$.

1) a) Calculate the mechanical energy of the system [pendulum, Earth] at the instant of launching $\left(\mathrm{P}_{1}\right)$ at $\mathrm{A}_{0}$.
b) Determine the magnitude $\mathrm{V}_{1}$ of the velocity $\overrightarrow{\mathrm{V}}_{1}$ of $\left(\mathrm{P}_{1}\right)$ just before colliding with $\left(\mathrm{P}_{2}\right)$.
2) a) Name the physical quantities that are conserved during this collision.
b) Show that the magnitude $V_{2}^{\prime}$ of the velocity $\vec{V}_{2}^{\prime}$ of $\left(\mathrm{P}_{2}\right)$, just after collision, is $8 \mathrm{~m} / \mathrm{s}$.

## B - Mechanical oscillator

A horizontal spring (S), of negligible mass and of stiffness $K=120 \mathrm{~N} / \mathrm{m}$, is connected at one of its ends B to a fixed support while the other end is attached to a ring $R$.
$\left(\mathrm{P}_{2}\right)$ moves on the horizontal path AB until it hits the ring R at point O ; $\left(\mathrm{P}_{2}\right)$ sticks to R forming a solid $(\mathrm{P})$, considered as a particle, of mass $\mathrm{m}=1.2 \mathrm{~kg}$. Thus $(\mathrm{P})$ and the spring $(\mathrm{S})$ form a horizontal mechanical oscillator of center of inertia G ; G moves without friction on a horizontal axis x ' Ox along AB . Just after collision and at the initial instant $\mathrm{t}_{0}=0$, G coincides with O , the equilibrium position of $(\mathrm{P})$, and has a velocity $\overrightarrow{\mathrm{V}}_{0}=\mathrm{V}_{0} \overrightarrow{\mathrm{i}}$ with $\mathrm{V}_{0}=2 \mathrm{~m} / \mathrm{s}$.
At an instant $t$, the abscissa of $G$ is $x$ and the algebraic value of its velocity is $v=\frac{d x}{d t}$.

1) Write down the expression of the mechanical energy of the system (oscillator, Earth) at an instant $t$, in terms of $\mathrm{K}, \mathrm{m}, \mathrm{x}$ and v .
2) Derive the differential equation in $x$ that describes the motion of $G$ and deduce the nature of its motion.
3) Knowing that the solution of this differential equation is $x=X_{m} \cos \left(\sqrt{\frac{K}{m}} t+\varphi\right)$, determine the values of the constants $X_{m}$ and $\varphi$.

## Second exercise: (7 points)

## Determination of the characteristics of a coil and a capacitor

The aim of this exercise is to determine the characteristics of a capacitor and a coil.
In order to determine these characteristics, we connect in series a capacitor of capacitance $C$, a coil of inductance $L$ and of resistance $r$, a resistor of resistance $\mathrm{R}=20 \Omega$ and a low frequency generator (LFG) delivering an alternating sinusoidal voltage $u$ of constant maximum value $U_{m}$ and of adjustable frequency $f$.
The circuit thus formed, carries an alternating sinusoidal current i (Fig. 1).
An oscilloscope is connected to display the voltage $\mathrm{u}=\mathrm{u}_{\mathrm{AM}}$ across the terminals of the (LFG) on channel $\left(\mathrm{Y}_{1}\right)$ and the voltage $\mathrm{u}_{\mathrm{BM}}$ across the terminals of the
 resistor ( R ) on channel $\left(\mathrm{Y}_{2}\right)$.
The settings of the oscilloscope are:
horizontal sensitivity: $\mathrm{S}_{\mathrm{h}}=2 \mathrm{~ms} /$ div;
vertical sensitivity: - On $\left(\mathrm{Y}_{1}\right): \mathrm{S}_{\mathrm{V} 1}=2 \mathrm{~V} / \mathrm{div}$;

$$
-\mathrm{On}\left(\mathrm{Y}_{2}\right): \mathrm{S}_{\mathrm{V} 2}=0.25 \mathrm{~V} / \mathrm{div} .
$$

A - For a given value $f_{0}$ of the frequency $f$ we observe on the screen of the oscilloscope the waveforms represented by figure 2 .

1) Determine $f_{o}$ and the proper angular frequency $\omega_{0}$.
2) Determine the maximum value $U_{m}$ of $u$ and the maximum current $I_{m}$ of $i$.
3) a) The waveforms show that a physical phenomenon takes place in the circuit. Name this phenomenon. Justify.
b) Deduce the relation between L and C .
4) The circuit between $A$ and $M$ is equivalent to a resistor of resistance $R_{t}=R+r$. Determine $R_{t}$ and deduce $r$.

B - The coil in the circuit of figure 1 is replaced by a resistor $\mathrm{r}_{1}$ of resistance $r_{1}=60 \Omega$ (figure 3 ).
The voltage across the terminals of the generator is $u=u_{A M}=U_{m} \cos \omega_{0} \mathrm{t}$. On the screen of the oscilloscope, we observe the waveforms represented by figure 4 . The settings of the oscilloscope are not changed.

1) Using the waveforms of figure 4 :
a) tell why the voltage $u_{\text {AM }}$ lags behind $u_{B M}$;
b) calculate the phase difference $\varphi$ between $u_{\mathrm{AM}}$ and $u_{\mathrm{BM}}$;


Fig. 2

c) determine the expressions of $u_{B M}$ and of $u_{A M}$ as a function of time $t$.
2) Write down the expression of $i$ as a function of time $t$.
3) The voltage across the terminal of the capacitor is:
$\mathrm{u}_{\mathrm{C}}=\mathrm{u}_{\mathrm{AD}}=\frac{8.9 \times 10^{-5}}{\mathrm{C}} \sin \left(125 \pi \mathrm{t}+\frac{\pi}{4}\right) ;[\mathrm{u}$ in V and t in s$]$.
By applying the law of addition of voltages and giving $t$ a particular value, determine the value of C .
$\mathbf{C}$ - Use the relation found in part [A-3 (b)] , calculate L.


Fig. 4

## Third exercise: (6 points)

## Dating by Carbon 14

The radioactive carbon isotope ${ }_{6}^{14} \mathrm{C}$ is a $\beta^{-}$emitter. In the atmosphere, ${ }_{6}^{14} \mathrm{C}$ exists with the carbon 12 in a constant ratio.
When an organism is alive it absorbs carbon dioxide that comes indifferently from carbon 12 and carbon 14. Just after the death of an organism, this absorption stops and carbon 14, that it has, disintegrate with a half life $\mathrm{T}=5700$ years.
In living organisms, the ratio of the number of carbon 14 atoms to that of the number of carbon 12 atoms is: $\mathrm{r}_{0}=\frac{\text { initial number of carbon } 14 \text { atoms }}{\text { number of carbon } 12 \text { atoms }}=\frac{\mathrm{N}_{\mathrm{o}}\left({ }^{14} \mathrm{C}\right)}{\mathrm{N}^{\prime}\left({ }^{12} \mathrm{C}\right)}=10^{-12}$.
After the death of an organism by a time $t$, the ratio of the number of carbon 14 atoms to that of the number of carbon 12 atoms becomes: $\mathrm{r}=\frac{\text { remaining number of carbon } 14 \text { atoms }}{\text { number of carbon } 12 \text { atoms }}=\frac{\mathrm{N}\left({ }^{14} \mathrm{C}\right)}{\mathrm{N}^{\prime}\left({ }^{12} \mathrm{C}\right)}$.

1) The disintegration of ${ }_{6}^{14} \mathrm{C}$ is given by: ${ }_{6}^{14} \mathrm{C} \rightarrow{ }_{\mathrm{Z}}^{\mathrm{A}} \mathrm{N}+\beta^{-}+{ }_{0}^{0} \mathrm{U}$.

Calculate Z and A , specifying the laws used.
2) Calculate, in year ${ }^{-1}$, the radioactive constant $\lambda$ of carbon 14 .
3) Using, the law of radioactive decay of carbon $14, N\left({ }^{14} \mathrm{C}\right)=\mathrm{N}_{\mathrm{o}}\left({ }^{14} \mathrm{C}\right) \times \mathrm{e}^{-\lambda t}$.

Show that $r=r_{0} e^{-\lambda t}$.
4) Measurements of $\frac{r}{r_{0}}$, for specimens $\mathrm{a}, \mathrm{b}$ and c , are given in the following table:

| ratio | specimen a | specimen b | specimen c |
| :---: | :---: | :---: | :---: |
| $\frac{\mathrm{r}}{\mathrm{r}_{0}}$ | 0.914 | 0.843 | 0.984 |

a) Specimen $b$ is the oldest. Why?
b) Determine the age of specimen $b$.
5) a) Calculate the ratio $\frac{\mathrm{r}}{\mathrm{r}_{0}}$ for $\mathrm{t}_{0}=0, \mathrm{t}_{1}=2 \mathrm{~T}, \mathrm{t}_{2}=4 \mathrm{~T}$ and $\mathrm{t}_{3}=6 \mathrm{~T}$.
b) Trace then the curve $\frac{r}{r_{0}}=f(t)$ by taking the following scales:

- On the abscissa axis: $1 \mathrm{~cm} \rightarrow 2 \mathrm{~T}$
- On the ordinate axis: $1 \mathrm{~cm} \rightarrow \frac{\mathrm{r}}{\mathrm{r}_{0}}=0.2$
c) To determine the date of death of a living organism, it is just enough to measure $\frac{r}{r_{0}}$.

Explain why we cannot use the traced curve to determine the date of the death of an organism that died several millions years ago.

|  | امتحانـات الثشهادة الثلانويـة العامة الفرع : علوم الحياة | وزارة التربيةّ والتّعليم العالثي المديرية العامـة للتربية دائرة الامتحاتـات |
| :---: | :---: | :---: |
| الالرقم: | مسابقة في مادة الفيزياء المدة ساعتان | مشروع مـيار التصحيح |

## Solutions

## First exercise ( 7 points)

| Part of the $\mathbf{Q}$ | Answer | Mark |
| :---: | :---: | :---: |
| A-1-a | $\mathrm{ME}_{\mathrm{i}}=\mathrm{KE}_{\mathrm{i}}+\mathrm{PEg}_{\mathrm{i}}=1 / 2 \mathrm{~m}_{1} \mathrm{~V}_{\mathrm{i}}^{2}+\mathrm{m}_{1} \mathrm{~g} \ell=0.5 \times 0.2 \times 64+0.2 \times 10 \times 1.8=10 \mathrm{~J}$ | 0.75 |
| A-1-b | Since there is no friction then ME is conserved so $\begin{aligned} & \mathrm{ME}_{\mathrm{i}}=10=\mathrm{ME}_{\mathrm{A}}=\frac{1}{2} \mathrm{~m}_{1} \mathrm{~V}_{1}^{2}+0 \\ & \Rightarrow 10=0.1 \mathrm{~V}_{1}^{2}+0 \Rightarrow \mathrm{~V}_{1}=10 \mathrm{~m} / \mathrm{s} \end{aligned}$ | 0.75 |
| A-2.a | The linear momentum and the kinetic energy. | 0.50 |
| A-2.b | Conservation of linear momentum: $m_{1} \overrightarrow{V_{1}}+0=m_{1} \overrightarrow{V_{1}^{\prime}}+m_{2} \overrightarrow{V_{2}^{\prime}}$ but no deviation (head-on) $\begin{align*} & \Rightarrow \mathrm{m}_{1} \mathrm{~V}_{1}+0=\mathrm{m}_{1} \mathrm{~V}_{1}^{\prime}+\mathrm{m}_{2} \mathrm{~V}_{2}^{\prime} \Rightarrow \mathrm{m}_{1}\left(\mathrm{~V}_{1}-\mathrm{V}_{1}^{\prime}\right)=\mathrm{m}_{2} \mathrm{~V}_{2}^{\prime} \ldots \\ & \text { collision is elastic: } 1 / 2 \mathrm{~m}_{1} \mathrm{~V}_{1}^{2}=1 / 2 \mathrm{~m}_{1}\left(\mathrm{~V}_{1}^{\prime}\right)^{2}+1 / 2 \mathrm{~m}_{2}\left(\mathrm{~V}_{2}^{\prime}\right)^{2} \\ & \Rightarrow \mathrm{~m}_{1}\left[\mathrm{~V}_{1}^{2}-\left(\mathrm{V}_{1}^{\prime}\right)^{2}\right]=\mathrm{m}_{2}\left(\mathrm{~V}_{2}^{\prime}\right)^{2} \quad(2)  \tag{2}\\ & \text { Divide (2) by (1) we get: } \mathrm{V}_{1}+\mathrm{V}_{1}^{\prime}=\mathrm{V}_{2}^{\prime} \tag{3} \end{align*}$ <br> Equations (1) and (3) give: $\mathrm{V}_{2}^{\prime}=8 \mathrm{~m} / \mathrm{s}$. | 1.5 |
| B-1 | M.E $=1 / 2 \mathrm{kx}^{2}+1 / 2 \mathrm{mV}^{2}$. | 0.5 |
| B-2 | $\begin{aligned} & \text { M.E }=\text { constant } \\ & \Rightarrow \frac{\mathrm{dM} \cdot \mathrm{E}}{\mathrm{dt}}=0 \\ & \Rightarrow \mathrm{kxx}^{\prime}+\mathrm{mVV}^{\prime}=0 ; \mathrm{V}=\mathrm{x}^{\prime} \neq 0 \text { and } \mathrm{V}^{\prime}=\mathrm{x}^{\prime \prime} \\ & \Rightarrow \mathrm{x}^{\prime \prime}+\left(\frac{\mathrm{K}}{\mathrm{~m}}\right) \mathrm{x}=0 . \end{aligned}$ <br> This differential equation has the form of $\mathrm{x}^{\prime \prime}+\omega_{0}^{2} \mathrm{x}=0$; The motion is simple harmonic. | 1 |
| B-3 | $\begin{aligned} & \mathrm{ME}_{\mathrm{x}=0}=\mathrm{ME}_{\mathrm{x}=\mathrm{xm}} \Rightarrow \frac{1}{2} \mathrm{mV}_{\mathrm{o}}^{2}+\frac{1}{2} \mathrm{Kx}_{\mathrm{o}}^{2}=\frac{1}{2} \mathrm{KX}_{\mathrm{m}}^{2} \\ & \frac{1}{2} \times 1.2 \times 2^{2}+0=\frac{1}{2} \times 120 \times \mathrm{X}_{\mathrm{m}}^{2} \Rightarrow \mathrm{X}_{\mathrm{m}}=0.2 \mathrm{~m}=20 \mathrm{~cm} . \\ & \mathrm{x}=\mathrm{X}_{\mathrm{m}} \cos \left(\sqrt{\frac{\mathrm{~K}}{\mathrm{~m}}} \mathrm{t}+\varphi\right) \end{aligned}$ <br> at $\mathrm{t}=0 \mathrm{~s}, \mathrm{x}=0 \Rightarrow 0=\mathrm{X}_{\mathrm{m}} \cos \varphi \Rightarrow \cos \varphi=0 \Rightarrow \varphi= \pm \frac{\pi}{2}$ but at $\mathrm{t}=0$ we have $\mathrm{v}=\mathrm{V}_{\mathrm{o}}=-\mathrm{X}_{\mathrm{m}} \sin \varphi>0 \Rightarrow \varphi=-\frac{\pi}{2} \mathrm{rd}$ | 1 1 |

## Second exercise (7 points)

| Part of the $\mathbf{Q}$ | Answer | Mark |
| :---: | :---: | :---: |
| A-1 | $\mathrm{T}_{\mathrm{o}}=8 \times 2=16 \mathrm{~ms} \Rightarrow \mathrm{f}_{\mathrm{o}}=\frac{1}{\mathrm{~T}_{o}}=62.5 \mathrm{~Hz}$ and $\omega_{\mathrm{o}}=2 \pi \mathrm{f}_{\mathrm{o}}=125 \pi \mathrm{rd} / \mathrm{s}$. | $\begin{array}{\|c\|c\|} \hline 0.5 ; 0.25 \\ 0.25 \end{array}$ |
| A-2 | $\begin{aligned} & \mathrm{U}_{\mathrm{m}}=2 \times 2=4 \mathrm{~V} \\ & \mathrm{U}_{\mathrm{Rm}}=4 \times 0.25=1 \mathrm{~V} \Rightarrow \mathrm{I}_{\mathrm{m}}=\frac{\mathrm{U}_{\mathrm{Rm}}}{\mathrm{R}}=\frac{1}{20}=0.05 \mathrm{~A} \end{aligned}$ | $\begin{gathered} \hline 0.25 \\ 0.75 \end{gathered}$ |
| A-3-a | Current resonance, since $u_{\text {AM }}$ and $u_{B M}=R i$ are in phase | 0.25;0.25 |
| A-3-b | Since we have current resonance then $\mathrm{LC} \omega_{0}^{2}=1$ so $\mathrm{LC}=6.49 \times 10^{-6}$. | 0.25; 0.5 |
| A-4 | $\mathrm{U}_{\mathrm{m}}=\mathrm{I}_{\mathrm{m}} \times \mathrm{R}_{\mathrm{t}} \Rightarrow \mathrm{R}_{\mathrm{t}}=\frac{4}{0.05}=80 \Omega \Rightarrow \mathrm{r}=80-20=60 \Omega$ | 0.25; 0.25 |
| 5- B-1-a | Since $u_{\text {BM }}$ reaches its maximum before that of $u_{\text {AM }}$. | 0.25 |
| B-1-b | $2 \pi \mathrm{rd} \rightarrow 8 \mathrm{div} \rightarrow \mathrm{T}_{0}$ $\varphi \rightarrow 1 \operatorname{div} \Rightarrow \varphi=\frac{2 \pi}{8}=\frac{\pi}{4} \mathrm{rd}$. | 0.5 |
| B-1-c | $\begin{aligned} & \mathrm{U}_{\mathrm{BM} \max }=2.8 \times 0.25=0.7 \mathrm{~V} \\ & \Rightarrow \mathrm{u}_{\mathrm{BM}}=0.7 \cos \left(125 \pi \mathrm{tt}+\frac{\pi}{4}\right) \quad\left(\mathrm{u}_{\mathrm{BM}} \text { in } \mathrm{V}, \mathrm{t} \text { in } \mathrm{s}\right) \\ & \mathrm{U}_{\mathrm{m}}=2 \times 2=4 \mathrm{~V} \Rightarrow \mathrm{u}=4 \cos 125 \pi \mathrm{t} \quad(\mathrm{u} \text { in } \mathrm{V}, \mathrm{t} \text { in } \mathrm{s}) . \end{aligned}$ | $\begin{aligned} & 0.50 \\ & 0.25 \end{aligned}$ |
| B-2 | $\begin{aligned} & \mathrm{I}_{\mathrm{m}}=\frac{\text { UBMmex }^{R}}{\mathrm{R}}=\frac{2.8 \times 0.25}{20}=0.035 \mathrm{~A} \\ & \left.\Rightarrow \mathrm{i}=0.035 \cos \left(125 \pi t+\frac{\pi}{4}\right) \quad \text { (i in A, } \mathrm{t} \text { in } \mathrm{s}\right) . \end{aligned}$ | 0.5 |
| B-3 | The law of addition of voltages gives : $u_{A M}=u_{A D}+u_{D B}+u_{B M}$ $\begin{aligned} & 4 \cos 125 \pi t=\frac{8.9 \times 10^{-5}}{\mathrm{C}} \sin \left(125 \pi \mathrm{t}+\frac{\pi}{4}\right)+80 \times 0.035 \cos \left(125 \pi t+\frac{\pi}{4}\right) \\ & \text { For } 125 \pi \mathrm{t}=\frac{\pi}{2} ; 0=\frac{8.9 \times 10^{-5}}{\mathrm{C}} \cos \frac{\pi}{4}-2.8 \sin \frac{\pi}{4} \Rightarrow \frac{8.9 \times 10^{-5}}{\mathrm{C}}=2.8 \\ & \mathrm{C}=\frac{8.9 \times 10^{-5}}{2.8}=32 \times 10^{-6} \mathrm{~F}=32 \mu \mathrm{~F} . \end{aligned}$ | 1 |
| C | $\mathrm{LC}=6.49 \times 10^{-6} \Rightarrow \mathrm{~L} \times 32 \times 10^{-6}=6.49 \times 10^{-6} \Rightarrow \mathrm{~L}=\frac{6.49}{32}=0.2 \mathrm{H}$ | 0.25 |

## Third exercise ( 6 points)

| Part of the $\mathbf{Q}$ | Answer | Mark |
| :---: | :---: | :---: |
| 1 | ${ }_{6}^{14} \mathrm{C} \rightarrow{ }_{-1}^{0} \mathrm{e}+{ }_{\mathrm{Z}}^{\mathrm{A}} \mathrm{X}+{ }_{0}^{0} \mathrm{~V}$ law of conservation of mass number: <br> $14=0+\mathrm{A}+0$ then $\mathrm{A}=14$ <br> law of conservation of charge number: $6=0-1+Z+0$ then $Z=7$. | $\begin{gathered} 0.25 \\ 0.25 \\ \mathbf{0 . 2 5} \\ \mathbf{0 . 2 5} \end{gathered}$ |
| 2 | $\lambda=\frac{0.693}{\mathrm{~T}}=1.216 \times 10^{-4} \mathrm{year}^{-1}$ | 0.75 |
| 3 | $\mathrm{r}=\frac{\mathrm{N}\left({ }^{14} \mathrm{C}\right)}{\mathrm{N}^{\prime}\left({ }^{12} \mathrm{C}\right)}=\frac{\left.\mathrm{N}_{\mathrm{o}}{ }^{14} \mathrm{C}\right) \times \mathrm{e}^{-\lambda t}}{\mathrm{~N}^{\prime}\left({ }^{12} \mathrm{C}\right)} \text { with } \mathrm{r}_{0}=\frac{\mathrm{N}_{\mathrm{o}}\left({ }^{14} \mathrm{C}\right)}{\mathrm{N}^{\prime}\left({ }^{12} \mathrm{C}\right)} \text {, we can write } \mathrm{r}=\mathrm{r}_{0} \mathrm{e}^{-\lambda \mathrm{t}} .$ | 0.75 |
| 4-a | $\frac{r}{r_{0}}=e^{-\lambda t} \text { as } t \text { increases then } e^{-\lambda t} \text { decreases then } \frac{r}{r_{0}} \text { decreases }$ <br> Since specimen $b$ has the lowest ratio then it is the oldest. | 0.5 |
| 4-b | $\frac{r}{r_{0}}=e^{-\lambda t}=0.843$ then $\ln 0.843=-\lambda \times t$ thus the age of the specimen is $\mathrm{t}=\frac{-0.171}{-1.216 \times 10^{-4}}=1406.25$ years. | 1 |
| 5-a | the ratio $\frac{r}{r_{0}}=e^{-\lambda t}$ fot $t_{0}=0 \frac{r}{r_{0}}=1$; for $\mathrm{t}=2 \mathrm{~T}$ then $\frac{\mathrm{r}}{\mathrm{r}_{0}}=0.25$; for $\mathrm{t}=4 \mathrm{~T}$ then $\frac{\mathrm{r}}{\mathrm{r}_{0}}=0.0625$ for $\mathrm{t}=6 \mathrm{~T}$ then $\frac{\mathrm{r}}{\mathrm{r}_{0}}=0.015625$. | 1 |
| 5-b |  | 0.5 |
| 5-c | Since after millions of years the ratio $\frac{\mathrm{r}}{\mathrm{r}_{0}}$ becomes zero so we cannot determine the age of such organism. | 0.5 |

مسابقة في مادة الفيزياء الاء الاسم:

## This exam is formed of three exercises in three pages numbered from 1 to 3 <br> The use of non-programmable calculator is recommended

## First exercise: (7 points)

## Charging of a capacitor

In order to charge a capacitor, we connect up the series circuit that is represented in figure 1 . This circuit is formed of:

- a generator of constant e.m.f E and of negligible internal resistance;
- a resistor of resistance R ;
- a capacitor of capacitance $\mathrm{C}=1 \mu \mathrm{~F}$;
- a switch K.

The capacitor is initially neutral. At the instant $t_{0}=0$, we close $K$. At an instant $t$, the armature A carries a charge $q$ and the circuit is traversed by a current i whose direction is shown on the circuit.


Fig. 1

A - Analytical study

1) Write the expression of $q$ in terms of $u_{A B}$ and $C$.
2) Derive the differential equation that governs the variation of $q$ as a function of time.
3) Using the differential equation, deduce:
a) that the expression of the current at the instant $t_{0}=0$ is $I_{0}=\frac{E}{R}$;
b) the expression, of the maximum value $\mathrm{Q}_{\mathrm{m}}$ of q in terms of C and E .

## B - Exploitation of the curve

The variation of the charge q , as a function of time, is represented by the curve of figure 2. The straight line (OM) represents the tangent to the curve at the instant $\mathrm{t}_{0}=0$.
Using figure 2 :

1) a) indicate the maximum value $Q_{m}$ of $q$;
b) deduce the value of E .
2) a) Show that the value of $I_{0}$ is 1 mA ;
b) deduce the value of $R$.
3) Determine the values of $u_{A B}$ and of $i$ at the instant $\mathrm{t}_{1}=10^{-2} \mathrm{~s}$.


C - Energy stored in the capacitor
Fig. 2
Knowing that the energy stored in the
capacitor at an instant t is given by $\mathrm{w}=\frac{1}{2} \frac{\mathrm{q}^{2}}{\mathrm{C}}$, determine:

1) the values of $w$ at $t_{0}=0$ and at $t_{1}=10^{-2} \mathrm{~s}$;
2) the average electric power received by the capacitor between $t_{0}$ and $t_{1}$.

## Second exercise: (6 points)

## Measurement of the mass of an astronaut

The aim of this exercise is to measure, in a spaceship, the mass of an astronaut using a horizontal mechanical oscillator.

## A - Theoretical study

Consider a horizontal mechanical oscillator formed of a solid (S), of mass M, connected to two identical springs of negligible mass and each of stiffness $\mathrm{k}_{1}$. The center of inertia G of ( S ) may slide along a horizontal axis $\mathrm{x}^{\prime} \mathrm{Ox}$, where O is
 confounded with the equilibrium position of $G$.
At equilibrium, the two springs are neither compressed nor elongated (Fig.1).
The solid (S), is displaced by a distance $\mathrm{x}_{0}$ from its equilibrium position in the chosen positive direction, then released without initial velocity at the instant $t_{0}=0$. At an instant $t$, the abscissa of $G$ is $x$ and the algebraic value of its velocity $\vec{v}$ is $v=\frac{d x}{d t}=x^{\prime}$.
Neglect all frictional forces, and take the horizontal plane passing through $G$ as a gravitational potential energy reference.

1) Show that the expression of the elastic potential energy of the system [(S), two springs, Earth] is P. $\mathrm{E}_{\mathrm{e}}=\frac{1}{2} \mathrm{kx}^{2}$ where $\mathrm{k}=2 \mathrm{k}_{1}$.
2) Write, as a function of $k, M, v$ and $x$, at an instant $t$, the expression of the mechanical energy of the system [(S), two springs, Earth].
3) Derive the differential equation, in $x$, which describes the motion of $G$.
4) The solution of this differential equation is of the form: $x=A \cos \left(\omega_{0} t+\varphi\right)$ where $A, \omega_{0}$ and $\varphi$ are constants.
Determine the expressions of A and $\omega_{0}$ in terms of $\mathrm{x}_{0}, \mathrm{M}$ and k and determine the value of $\varphi$.
5) Deduce, in terms of $M$ and $k_{1}$, the expression of the proper period $T_{0}$ of the oscillations of $G$.

## B - Practical study

In spaceships, astronauts measure their masses using a mechanical oscillator as the one above.
An astronaut sits in a chair attached to two identical massless
 springs each of stiffness $\mathrm{k}_{1}=700 \mathrm{~N} / \mathrm{m}$ forming a horizontal oscillator (Fig.2).
Let M be the total mass of the astronaut and the chair. With an appropriate device, we record the variation of the abscissa $x$ of the center of mass of the system [astronaut, chair, 2 springs] as a function of time (Fig.3).


1) Indicate:
a) the type of the observed oscillations;
b) the value of the pseudo-period T of these oscillations.
2) The pseudo-period $T$ is approximately equal to the proper period $\mathrm{T}_{0}$. Conclude.
3) Deduce the mass of the astronaut knowing that the mass of the chair is 6.5 kg .


Fig. 3

Third exercise: ( 7 points)

## Energy levels of the hydrogen atom and of the nickel nucleus

The aim of this exercise is to compare and study the energy levels of an atom and a nucleus.
Given: $\mathrm{h}=6.62 \times 10^{-34} \mathrm{~J} . \mathrm{s} ; \mathrm{c}=3 \times 10^{8} \mathrm{~ms}^{-1} ; 1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$.

| Energy diagram of the hydrogen atom | Energy diagram of the nickel nucleus ${ }^{60} \mathrm{Ni}$ |
| :---: | :---: |
|  |  |

## A - Comparison

Referring to the two diagrams:

1) Name the state corresponding to the energy $\mathrm{E}=0 \mathrm{in}$ :
a) the hydrogen atom;
b) the nickel nucleus.
2) Show that the transitions of the nickel nucleus are much more energetic than those of the hydrogen atom;
3) Show that the energies of the hydrogen atom and that of the nickel nucleus are quantized.

B - Hydrogen atom

1) The hydrogen atom is in the ground level. Determine the minimum energy needed to ionize this atom.
2) The Lyman series of the hydrogen atom corresponds to a downward transition to the level $\mathrm{n}=1$.

Determine the maximum wavelength $\lambda_{\mathrm{m}}$ of the emitted photons in this series.
3) We send, on a hydrogen atom, separately, three photons $a, b$ and $c$, whose energies are indicated in the table below, knowing that, in each case, the hydrogen atom is in the ground state.

| Photon | a | b | c |
| :---: | :---: | :---: | :---: |
| Energy in eV | 12.09 | 12.30 | 14.60 |

a) Specify the photons that are absorbed by the hydrogen atom.
b) Indicate the state of the hydrogen atom in each case.

## C - Nickel 60 nucleus

The cobalt isotope ${ }_{27}^{60} \mathrm{Co}$, used for the treatment of certain kinds of cancer, is a
$\beta^{-}$emitter. The daughter nickel nucleus is found in an excited state $\left({ }_{28}^{60} \mathrm{Ni}^{*}\right)$.

1) Write down the equation of the $\beta^{-}$decay of cobalt 60 .
2) Write down the equation of the downward transition of $\mathrm{Ni}^{*}$.
3) Using the energy diagram of the nickel nucleus, determine the maximum wavelength $\lambda_{\mathrm{m}}^{\prime}$ of the emitted photon due to the downward transition of the nickel nucleus from an excited state to the ground state.
4) Compare $\lambda_{m}^{\prime}$ and $\lambda_{m}$.


## First exercise: (7 points)

| Part of <br> the $Q$ | Answer | Mark |
| :---: | :---: | :---: |
| A. 1 | $\mathrm{q}_{\mathrm{A}}=\mathrm{Cu}_{\text {AB }}$. | 0.25 |
| A. 2 | $\mathrm{E}=\mathrm{u}_{\mathrm{AB}}+\mathrm{Ri} \quad$ where $\quad \mathrm{i}=\frac{\mathrm{dq}_{\mathrm{A}}}{\mathrm{dt}} \Rightarrow \mathrm{E}=\frac{\mathrm{q}_{\mathrm{A}}}{\mathrm{C}}+\mathrm{R} \frac{\mathrm{dq}_{\mathrm{A}}}{\mathrm{dt}}$. | 0.75 |
| A.3.a | At $\mathrm{t}=0 ; \mathrm{q}_{\mathrm{A}}=0$ and $\mathrm{i}=\frac{\mathrm{dq}_{\mathrm{A}}}{\mathrm{dt}}=\mathrm{I}_{0} \Rightarrow \mathrm{E}=\mathrm{RI}_{0} \Rightarrow \mathrm{I}_{0}=\frac{\mathrm{E}}{\mathrm{R}}$. | 0.75 |
| A.3.b | When $t$ increases indefinitely $q_{A}$ tends to a constant value $\Rightarrow \frac{\mathrm{dq}_{\mathrm{A}}}{\mathrm{dt}}=0 \Rightarrow \mathrm{Q}_{\mathrm{m}}=\mathrm{CE}$ | 0.50 |
| B.1-a | $\mathrm{Q}_{\mathrm{m}}=10^{-5} \mathrm{C}$ | 0.25 |
| B.1-b | $\mathrm{E}=\frac{\mathrm{Q}_{\mathrm{m}}}{\mathrm{C}}=\frac{10^{-5}}{10^{-6}}=10 \mathrm{~V}$ | 0.25 |
| B.2.a | The slope of the tangent to the curve at the point of abscissa $t=0$ is : $I_{0}=\frac{10^{-5}}{10^{-2}}=10^{-3} \mathrm{~A}$ | 0.75 |
| B.2.b | But $\mathrm{I}_{0}=\frac{\mathrm{E}}{\mathrm{R}} \Rightarrow \mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}_{0}}=\frac{10}{10^{-3}}=10^{4} \Omega$. | 0.50 |
| B. 3 | At the instant $\mathrm{t}=10^{-2} \mathrm{~s}$, we find graphically $\mathrm{q}_{\mathrm{A}}=0.63 \times 10^{-5} \mathrm{C}$ $\begin{aligned} & \Rightarrow \mathrm{u}_{\mathrm{AB}}=\frac{\mathrm{q}_{\mathrm{A}}}{\mathrm{C}}=6.3 \mathrm{~V} \\ & \mathrm{u}_{\mathrm{BN}}=\mathrm{E}-\mathrm{u}_{\mathrm{AB}}=10-6.3=3.7 \mathrm{~V} . \\ & \mathrm{i}=\frac{\mathrm{u}_{\mathrm{BN}}}{\mathrm{R}}=3.7 \times 10^{-4} \mathrm{~A} \end{aligned}$ | 1.50 |
| C. 1 | At $\mathrm{t}_{0}=0, \mathrm{q}_{0}=0 \Rightarrow \mathrm{w}_{0}=0$ $\text { At } \mathrm{t}_{1}=10^{-2} \mathrm{~s}, \mathrm{q}_{1}=0.63 \times 10^{-5} \mathrm{C} \Rightarrow \mathrm{w}_{1}=\frac{1}{2} \frac{\mathrm{q}_{1}^{2}}{\mathrm{C}}=0.198 \times 10^{-4} \mathrm{~J}$ | 1.00 |
| C. 2 | $\mathrm{P}_{\mathrm{m}}=\frac{\Delta \mathrm{w}}{\Delta \mathrm{t}}=0.198 \times 10^{-2} \mathrm{~W}$ | 0.50 |

## Second exercise: (7 points)

| Part of the $\mathbf{Q}$ | Answer | Mark |
| :---: | :---: | :---: |
| A. 1 | The potential energy stored in the spring elongated by x is P. $\mathrm{E}_{\mathrm{e}}(1)=1 / 2 \mathrm{k}_{1} \mathrm{X}^{2}$. <br> The potential energy stored in the spring compressed by x is $\mathrm{PE}_{\mathrm{e}}(2)=1 / 2 \mathrm{k}_{1} \mathrm{X}^{2}$. $\Rightarrow \mathrm{PE}_{\mathrm{e}}=\mathrm{P} \cdot \mathrm{E}_{\mathrm{e}}(1)+\mathrm{P} \cdot \mathrm{E}_{\mathrm{e}}(2)=1 / 2 \mathrm{kx}^{2} \text { with } \mathrm{k}=2 \mathrm{k}_{1} .$ | 0.75 |
| A. 2 | $\mathrm{ME}=1 / 2 \mathrm{mv}^{2}+1 / 2 \mathrm{kx}^{2}$ | 0.50 |
| A. 3 | $\begin{aligned} & \text { No friction, } \mathrm{ME}=\text { constant } \Rightarrow \frac{\mathrm{dME}}{\mathrm{dt}}=0, \\ & \Rightarrow \mathrm{mv}^{\prime}+\mathrm{kx} \mathrm{x}^{\prime}=0 \Rightarrow \mathrm{x}^{\prime \prime}+\frac{\mathrm{k}}{\mathrm{~m}} \mathrm{x}=0 \end{aligned}$ | 0.75 |
| A. 4 | $\dot{\mathrm{x}}=-\mathrm{A} \omega_{0} \sin \left(\omega_{0} \mathrm{t}\right)$ et $\mathrm{x}^{\prime \prime}=-\mathrm{A} \omega_{0}^{2} \cos \left(\omega_{0} \mathrm{t}\right)$. <br> By replacing each term by its value, we obtain: $\Rightarrow \omega_{0}^{2}-\frac{\mathrm{k}}{\mathrm{~m}}=0 \Rightarrow \omega_{0}=\sqrt{\frac{\mathrm{k}}{\mathrm{~m}}} .$ <br> For $t=0: x=x_{0}$ and $v_{0}=0 \Rightarrow x_{0}=A \cos \varphi$ and $\mathrm{v}_{0}=-\mathrm{A} \cdot \sin \varphi=0$; <br> $\Rightarrow \varphi=0$ or $\pi$ or $x_{0}>0 \Rightarrow \varphi=0$ and $x_{0}=A$ | 1.50 |
| A. 5 | The proper period : $\mathrm{T}_{0}=\frac{2 \pi}{\omega_{0}}=2 \pi \sqrt{\frac{\mathrm{M}}{\mathrm{k}}}=\pi \sqrt{\frac{2 \mathrm{M}}{\mathrm{k}_{1}}}$ | 0.50 |
| B.1-a | Free damped oscillations. | 0.25 |
| B.1-b | The period $\mathrm{T}=1.5 \mathrm{~s}$. | 0.50 |
| B. 2 | $\mathrm{T}=\mathrm{T}_{0}$ because of small damping | 0.25 |
| B. 3 | $\mathrm{T}_{0}=\pi \sqrt{\frac{2 \mathrm{M}}{\mathrm{k}_{1}}} \Rightarrow \mathrm{M}=\frac{\mathrm{k}_{1} \mathrm{~T}_{0}^{2}}{2 \pi^{2}} \Rightarrow \mathrm{M}=79.87 \mathrm{~kg} .$ <br> The mass of the astronaut is: $\mathrm{M}_{\mathrm{A}}=79.87-6.5=73.37 \mathrm{~kg}$. | 1.00 |

## Third exercise: ( 6 points)

| Part of the $\mathbf{Q}$ | Answer | Mark |
| :---: | :---: | :---: |
| A.1.a | For an atom: $\mathrm{E}=0$ corresponds to the ionized state | 0.25 |
| A.1.b | For a nucleus: $\mathrm{E}=0$ corresponds to the ground state. | 0.25 |
| A. 2 | For an atom, the energy changes are of the order of eV . For a nucleus, the energy changes are of the order of MeV . | 0.50 |
| A. 3 | Both have specific values of energy. | 0.25 |
| B. 1 | $\mathrm{E}_{\text {min }}=\mathrm{E}_{\infty}-\mathrm{E}_{1}=0+13.6=13.6 \mathrm{eV}$. | 0.50 |
| B. 2 | $\begin{aligned} & \mathrm{E}_{\mathrm{ph}}=\mathrm{E}_{\mathrm{n}}-\mathrm{E}_{1} \text { but } \mathrm{E}_{\mathrm{ph}}=\frac{\mathrm{hc}}{\lambda} \Rightarrow \mathrm{E}_{\mathrm{ph}(\min )} \text { when } \lambda_{\max } \text { then } \mathrm{n}=2 \text { so } \\ & \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda_{\max }}=\mathrm{E}_{2}-\mathrm{E}_{1}=(-3.4+13.6) \times 1.6 \times 10^{-19} \end{aligned}$ <br> thus $\lambda_{\mathrm{m}}=1.216 \times 10^{-7} \mathrm{~m}$. | 0.75 |
| B.3.a | $\mathrm{E}_{\mathrm{ph}}=\mathrm{E}_{\mathrm{n}}-\mathrm{E}_{1}$ then $\mathrm{E}_{\mathrm{n}}=\mathrm{E}_{\mathrm{ph}}+\mathrm{E}_{1}$ <br> for photon (a): $\mathrm{E}_{\mathrm{ph}}=12.09 \Rightarrow-13.6+12.09=-1,51 \mathrm{eV}=\mathrm{E}_{3}$ <br> $\mathrm{E}_{\mathrm{n}}=-1.51 \mathrm{eV}=\mathrm{E}_{3}$ so this photon is absorbed; <br> for photon (b): $\mathrm{E}_{\mathrm{ph}}=12.30 \Rightarrow-13.6+12.09 \Rightarrow \mathrm{E}_{\mathrm{n}}=-1.3 \mathrm{eV} \neq \mathrm{E}_{\mathrm{n}}$ so this photon cannot be absorbed; <br> for photon (c) $\mathrm{E}_{\mathrm{ph}}=14.6>\mathrm{E}_{1}$ so this is absorbed. | 1.50 |
| B.3.b | For photon (a): $2^{\text {nd }}$ excited state $\mathrm{E}_{3}$; for photon (b); ground state for photon (c): ionized state. | 0.75 |
| C. 1 | ${ }_{27}^{60} \mathrm{Co} \rightarrow{ }_{-1}^{0} \mathrm{e}+{ }_{28}^{60} \mathrm{Ni}^{*}$ | 0.50 |
| C. 2 | ${ }_{28}^{60} \mathrm{Ni}^{*} \rightarrow{ }_{28}^{60} \mathrm{Ni}+\gamma$. | 0.25 |
| C. 3 | $\lambda_{\mathrm{m}}{ }^{\mathrm{c}}$ corresponds to the smallest energy: $\begin{aligned} & \Delta \mathrm{E}=1.33-0=1.33 \mathrm{MeV}=2.128 \times 10^{-13} \mathrm{~J} \\ & \lambda_{\mathrm{m}}^{\prime}=\frac{\mathrm{h} . \mathrm{C}}{\Delta \mathrm{E}}=9.33 \times 10^{-13} \mathrm{~m} \end{aligned}$ | 1.00 |
| C. 4 | $\frac{\lambda_{\mathrm{m}}^{\prime}}{\lambda_{\mathrm{m}}}=7,6 \times 10^{-6} \Rightarrow \lambda_{\mathrm{m}}^{\prime} \lll \ll \lambda_{\mathrm{m}}$ | 0.50 |

