

الاسم:  
الرقم:

مسابقة في مادة الفيزياء  
المدة: ساعتان

**This exam is formed of three obligatory exercises in three pages.**  
**The use of non-programmable calculators is recommended.**

**Exercise 1 (7 points)**

**Horizontal elastic pendulum**

A mechanical oscillator is formed by a block (S) of mass  $m$  and a spring of negligible mass and spring constant  $k$ . (S) is attached to one end of the spring, and the other end of the spring is connected to a fixed support A. (S) can move without friction on a horizontal surface (Doc. 1).

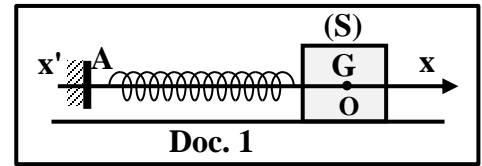
The aim of this exercise is to determine the values of  $m$  and  $k$ .

At equilibrium, the center of mass G of (S) coincides with the origin O of the  $x$ -axis.

(S) is displaced horizontally in the positive direction.

At the instant  $t_0 = 0$ , the abscissa of G is  $x_0$  and (S) is launched in the

negative direction with an initial velocity  $\vec{v}_0 = v_0 \vec{i}$  ( $v_0 < 0$ ) where  $\vec{i}$  is the unit vector of the  $x$ -axis.



At an instant  $t$ , the abscissa of G is  $x$  and the algebraic value of its velocity is  $v = x' = \frac{dx}{dt}$ .

The horizontal plane containing G is taken as a reference level for gravitational potential energy.

- 1) Write, at an instant  $t$ , the expression of the mechanical energy of the system (Oscillator, Earth) in terms of  $x$ ,  $m$ ,  $k$  and  $v$ .
- 2) Establish the second order differential equation in  $x$  that governs the motion of (S).
- 3) Deduce the expression of the proper angular frequency  $\omega_0$  of the oscillations in terms of  $m$  and  $k$ .
- 4) The solution of the obtained differential equation is:

$$x = X_m \sin(\omega_0 t + \varphi), \text{ where } X_m, \omega_0 \text{ and } \varphi \text{ are constants.}$$

Write the expression of  $v$  in terms of  $X_m$ ,  $\omega_0$ ,  $\varphi$  and  $t$ .

- 5) Write the expressions of  $x_0$  and  $v_0$  in terms of  $X_m$ ,  $\omega_0$  and  $\varphi$ .

- 6) Deduce that:  $X_m = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$ .

- 7) An appropriate device traces  $x$  and  $v$  as functions of time as shown in documents 2 and 3 respectively.

Referring to documents (2) and (3):

7-1) specify the type of the oscillations;

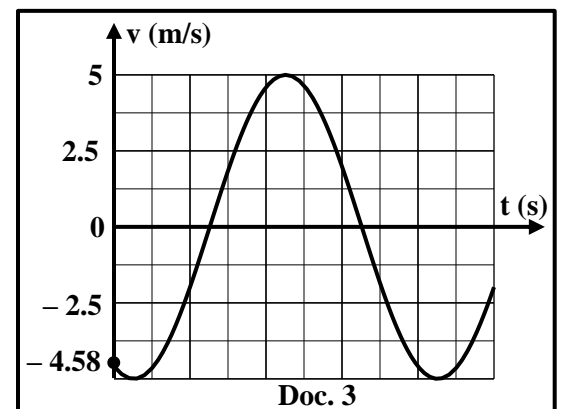
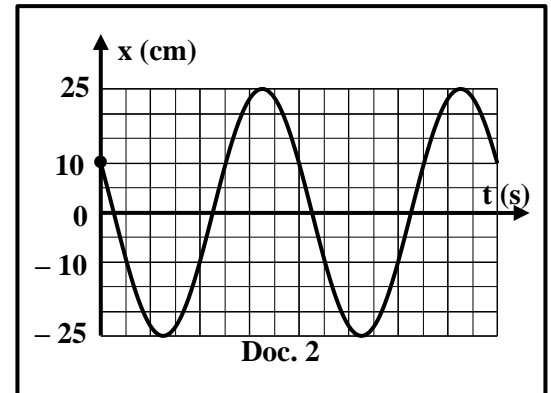
7-2) indicate the values of  $x_0$ ,  $v_0$ ,  $X_m$  and  $V_m$ , where  $V_m$  is the amplitude of  $v$ .

- 8) Deduce that  $\omega_0$  is approximately equal to 20 rad/s.
- 9) We repeat the same experiment by replacing the block (S) of mass  $m$  by another block (S') of mass  $m' = 0.8$  kg.

The new proper angular frequency is  $\omega' = \frac{\omega_0}{2}$ .

9-1) Write the expression of  $\omega'$  in terms of  $m'$  and  $k$ .

9-2) Deduce the values of  $k$  and  $m$ .



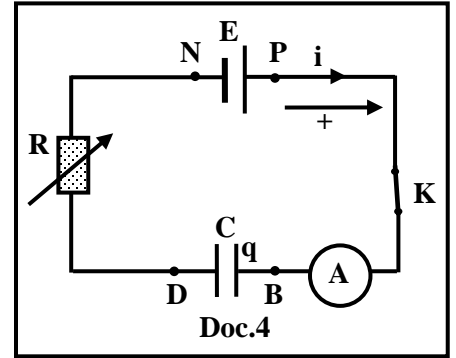
## Exercise 2 (7 points) Capacitance of a capacitor

The aim of this exercise is to determine the capacitance  $C$  of a capacitor. We set-up the series circuit of document 4.

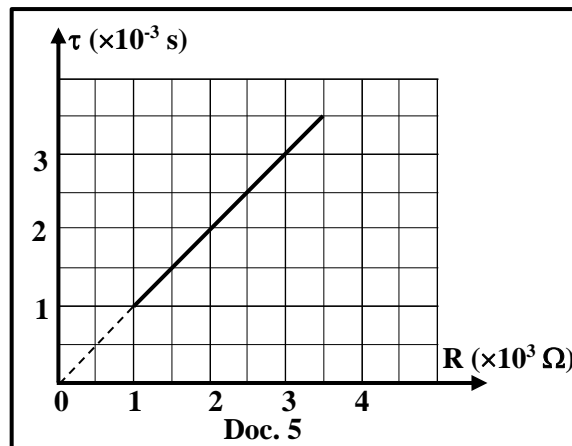
This circuit includes:

- an ideal battery of electromotive force  $E = 10 \text{ V}$ ;
- a rheostat of resistance  $R$ ;
- a capacitor of capacitance  $C$ ;
- an ammeter (A) of negligible resistance;
- a switch  $K$ .

Initially the capacitor is uncharged. We close the switch  $K$  at the instant  $t_0 = 0$ . At an instant  $t$ , plate B of the capacitor carries a charge  $q$  and the circuit carries a current  $i$  as shown in document 4.



- 1) Write the expression of  $i$  in terms of  $C$  and  $u_C$ , where  $u_C = u_{BD}$  is the voltage across the capacitor.
- 2) Establish the differential equation that governs the variation of  $u_C$ .
- 3) The solution of this differential equation is of the form:  $u_C = a + b e^{-\frac{t}{\tau}}$ . Determine the expressions of the constants  $a$ ,  $b$  and  $\tau$  in terms of  $E$ ,  $R$  and  $C$ .
- 4) Deduce that the expression of the current is:  $i = \frac{E}{R} e^{-\frac{t}{RC}}$ .
- 5) The ammeter (A) indicates a value  $I_0 = 5 \text{ mA}$  at  $t_0 = 0$ . Deduce the value of  $R$ .
- 6) Write the expression of  $u_R = u_{DN}$  in terms of  $E$ ,  $R$ ,  $C$  and  $t$ .
- 7) At an instant  $t = t_1$ , the voltage across the capacitor is  $u_C = u_R$ .
  - 7-1) Show that  $t_1 = RC \ln 2$ .
  - 7-2) Calculate the value of  $C$  knowing that  $t_1 = 1.4 \text{ ms}$ .
- 8) In order to verify the value of  $C$ , we vary the value of  $R$ . Document 5 represents  $\tau$  as a function of  $R$ .
  - 8-1) Show that the shape of the curve in document 5 is in agreement with the expression of  $\tau$  obtained in part 3.
  - 8-2) Using the curve of document 5, determine again the value of  $C$ .



### Exercise 3 (6 points)

### Aspects of Light

The aim of this exercise is to show evidence of the two aspects of light.

#### 1) First aspect

Consider Young's double-slit experiment. The two thin parallel horizontal slits  $S_1$  and  $S_2$  are separated by a distance  $a = 0.5$  mm.

The screen (E) is placed parallel to the plane of the slits at a distance  $D = 2$  m.

A laser source illuminates the two slits by a monochromatic light of wavelength  $\lambda = 600$  nm in air, under normal incidence.

O is the point of intersection between the perpendicular bisector of  $[S_1S_2]$  and the screen (E).

P is a point on the screen having an abscissa

$$x_P = \overline{OP} = 9.6 \text{ mm (Doc. 6).}$$

1-1) Calculate the inter-fringe distance  $i$ .

1-2) Specify the nature and the order of the fringe whose center is point P.

1-3) Slits  $S_1$  and  $S_2$  are replaced by a horizontal slit S of width  $a_1 = 0.1$  mm. O is the center of the central bright fringe and  $\alpha = 2\theta_1$  where  $\alpha$  is the angular width of the central bright fringe ( $\theta_1$  is a small angle) (Doc. 7).

1-3-1) Name the phenomenon that takes place at the slit S.

1-3-2) Show that the width  $L$  of the central bright fringe is given by the

$$\text{expression: } L = \frac{2\lambda D}{a_1}.$$

1-3-3) Deduce the distance  $d$  between O and the center of the first dark fringe.

1-3-4) Deduce that P is neither the center of a bright fringe nor the center of a dark fringe.

1-4) The previous two experiments show evidence of an aspect of light. Name this aspect.

#### 2) Second aspect

The monochromatic radiation of wavelength  $\lambda = 600$  nm in air, emitted by the laser source, illuminates now the surface of a lithium metal of work function  $W_0 = 2.39$  eV.

Given:

Planck's constant  $h = 6.6 \times 10^{-34}$  J.s ;  $1 \text{ eV} = 1.6 \times 10^{-19}$  J

Take: the speed of light in air  $c = 3 \times 10^8$  m/s.

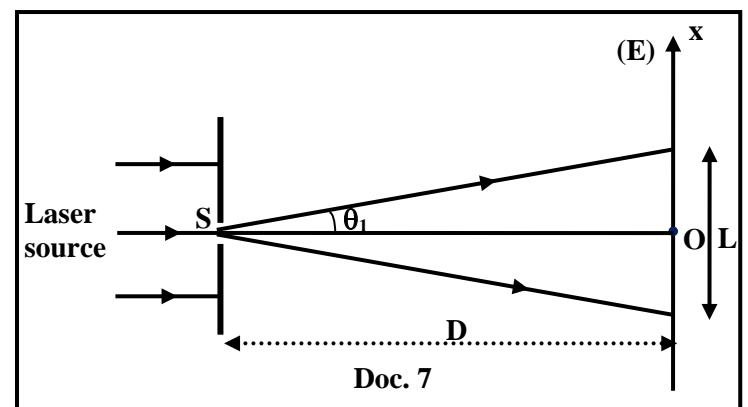
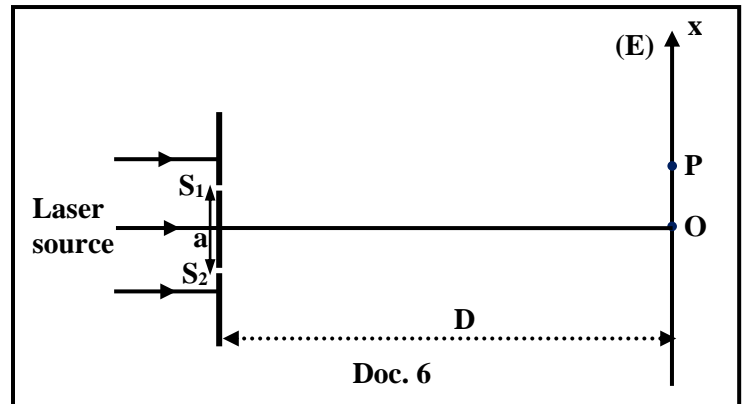
2-1) Define the work function (extraction energy) of a metal.

2-2) Calculate, in eV, the energy of a photon in this radiation.

2-3) Deduce that there is no photoelectric emission from the surface of the lithium metal.

2-4) In order to extract electrons from the surface of the lithium metal, the laser source is replaced by another one emitting a radiation of wavelength  $\lambda' = 500$  nm in air. Determine, in eV, the maximum kinetic energy of the liberated electrons.

2-5) This experiment shows evidence of an aspect of light. Name this aspect.



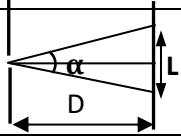
**Exercise 1 (7 points) Horizontal elastic pendulum**

| Partie | Answer                                                                                                                                                                                                                                                                                                        | Mark                   |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|
| 1      | $ME = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$                                                                                                                                                                                                                                                                      | 0.5                    |
| 2      | There is no friction therefore the mechanical energy is conserved.<br>ME = constant , then $\frac{dME}{dt} = 0$ , hence $m v v' + k x x' = 0$ with $v = x'$ and $v' = x''$<br>$x' (mv + kx) = 0$ , but $x' = 0$ is rejected ; therefore, $x'' + \frac{k}{m}x = 0$                                             | 1                      |
| 3      | The differential equation is of the form: $x'' + \omega_0^2 x = 0$<br>then : $\omega_0 = \sqrt{\frac{k}{m}}$                                                                                                                                                                                                  | 0.5                    |
| 4      | $v = X_m \omega_0 \cos(\omega_0 t + \varphi)$                                                                                                                                                                                                                                                                 | 0.25                   |
| 5      | $x_0 = X_m \sin\varphi$<br>$v_0 = \omega_0 X_m \cos\varphi$                                                                                                                                                                                                                                                   | 0.25<br>0.25           |
| 6      | $\sin\varphi = \frac{x_0}{X_m}$ and $\cos\varphi = \frac{v_0}{\omega_0 X_m}$<br>$\sin^2\varphi + \cos^2\varphi = 1$<br>$\frac{x_0^2}{X_m^2} + \frac{v_0^2}{\omega_0^2 X_m^2} = 1$ , so $X_m^2 = x_0^2 + \frac{v_0^2}{\omega_0^2}$ Therefore, $X_m = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$                  | 1                      |
| 7      | 7.1 Free undamped mechanical oscillations since the amplitude $x_m$ is constant                                                                                                                                                                                                                               | 0.5                    |
|        | 7.2 $x_0 = 10 \text{ cm}$ ; $v_0 = -4.58 \text{ m/s}$<br>$X_m = 25 \text{ cm}$ ; $V_m = 5 \text{ m/s}$                                                                                                                                                                                                        | 0.25 0.25<br>0.25 0.25 |
| 8      | Substituting the values of $x_0$ , $v_0$ and $X_m$ into the expression of $X_m$ gives :<br>$0.25 = \sqrt{0.1^2 + \frac{-4.58^2}{\omega_0^2}}$ , then $\omega_0 = 19.98 \cong 20 \text{ rad/s}$<br><u>Or :</u><br>$V_m = \omega_0 X_m$ , then $\omega_0 = \frac{V_m}{X_m} = \frac{5}{0.25} = 20 \text{ rad/s}$ | 0.5                    |
| 9      | 9.1 $\omega' = \sqrt{\frac{k}{m'}}$                                                                                                                                                                                                                                                                           | 0.25                   |
|        | 9.2 $\omega' = 10 \text{ rad/s}$<br>$k = m' \times \omega'^2 = 0.8 \times 10^2 = 80 \text{ N/m}$<br>$m = \frac{k}{\omega_0^2} = \frac{80}{400} = 0.2 \text{ kg}$                                                                                                                                              | 0.5<br>0.5             |

**Exercise 2 (7 points) Capacitance of a capacitor**

| Part | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | notes |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 1    | $i = \frac{dq}{dt}$ , but $q = C \times u_C$ , then $i = C \frac{du_C}{dt}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 0.5   |
| 2    | $E = u_{BD} + u_{DN} = u_C + Ri$ , but $i = C \frac{du_C}{dt}$ ; therefore, $E = u_C + RC \frac{du_C}{dt}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 0.75  |
| 3    | <p><math>\frac{du_C}{dt} = -\frac{b}{\tau} e^{-\frac{t}{\tau}}</math> ; Substituting <math>u_C</math> and <math>\frac{du_C}{dt}</math> in the differential equation gives :</p> <p><math>E = a + b e^{-\frac{t}{\tau}} + RC (-\frac{b}{\tau} e^{-\frac{t}{\tau}})</math> , so <math>E = a + b e^{-\frac{t}{\tau}} (1 - \frac{RC}{\tau})</math></p> <p>By comparison we obtain :</p> <p><math>a = E</math> and <math>b e^{-\frac{t}{\tau}} (1 - \frac{RC}{\tau}) = 0</math> , but <math>b e^{-\frac{t}{\tau}} = 0</math> is rejected ,then <math>1 - \frac{RC}{\tau} = 0</math></p> <p>Therefore, <math>\tau = RC</math></p> <p>At <math>t_0 = 0</math>, the charge is <math>q_0 = 0</math> , then <math>u_{C0} = 0</math>.</p> <p>Substituting <math>u_{C0} = 0</math> into the expression of <math>u_C</math> gives: <math>0 = a + b</math> , so <math>b = -a = -E</math></p> | 2     |
| 4    | $i = C \frac{du_C}{dt} = C \frac{E}{\tau} e^{-\frac{t}{\tau}} = \frac{E}{R} e^{-\frac{t}{\tau}}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 0.5   |
| 5    | At $t_0 = 0 : i = I_0 = \frac{E}{R} e^0$ , then $I_0 = \frac{E}{R}$ , thus $R = \frac{E}{I_0} = \frac{10}{5 \times 10^{-3}} = 2 \times 10^3 \Omega$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 0.5   |
| 6    | $u_R = Ri = RC \frac{du_C}{dt} = RC \frac{E}{\tau} e^{-\frac{t}{\tau}}$ , then $u_R = E e^{-\frac{t}{\tau}}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 0.5   |
| 7    | <p>7.1 <math>u_C = u_R</math></p> <p><math>E - E e^{-\frac{t_1}{\tau}} = E e^{-\frac{t_1}{\tau}}</math> , so <math>E = 2 E e^{-\frac{t_1}{\tau}}</math> , then <math>\frac{1}{2} = e^{-\frac{t_1}{\tau}}</math> , hence <math>-\ln 2 = -\frac{t_1}{\tau}</math></p> <p>Then, <math>t_1 = \tau \ln 2</math> ; therefore, <math>t_1 = RC \ln 2</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 0.75  |
|      | <p>7.2 <math>C = \frac{t_1}{R \ln 2} = \frac{1.4 \times 10^{-3}}{2 \times 10^3 \times \ln 2} = 1 \times 10^{-6} F</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 0.5   |
| 8    | <p>8.1 The curve is a straight line passing through the origin with a positive slope, then it is in agreement with the expression <math>\tau = RC</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 0.5   |
|      | <p>8.2 Slope = <math>C = \frac{\Delta \tau}{\Delta R} = \frac{3 \times 10^{-3}}{3 \times 10^3} = 1 \times 10^{-6} F</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 0.5   |

**Exercise 3 (6 points) Aspects of Light**

| Part  |                                                                                                      | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                            | Mark                                                                                |      |
|-------|------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|------|
| 1     | 1.1                                                                                                  | $i = \frac{\lambda D}{a} = \frac{600 \times 10^{-9} \times 2}{0.5 \times 10^{-3}} = 24 \times 10^{-4} \text{ m} = 2.4 \text{ mm}$                                                                                                                                                                                                                                                                                                                 | 0.5                                                                                 |      |
|       | 1.2                                                                                                  | $x_P = 9.6 \text{ mm} = 4 i$ , then P is the center of the 4 <sup>th</sup> bright fringe.<br><u>Or:</u><br>P is the center of a bright fringe if $x_P = \frac{k \lambda D}{a}$ with $k \in \mathbb{Z}$ .<br>$x_P = \frac{k \lambda D}{a}$ , then $k = \frac{a x_P}{\lambda D} = \frac{0.5 \times 10^{-3} \times 9.6 \times 10^{-3}}{600 \times 10^{-9} \times 2} = 4 \in \mathbb{Z}$ , then P is the center of the 4 <sup>th</sup> bright fringe. | 1                                                                                   |      |
|       | 1.3.1                                                                                                | Diffraction of light                                                                                                                                                                                                                                                                                                                                                                                                                              | 0.25                                                                                |      |
|       | 1.3.2                                                                                                | From the figure: $\tan \frac{\alpha}{2} = \frac{L/2}{D}$ , but $\alpha$ is small then $\tan \alpha \cong \alpha$<br>So $\frac{\alpha}{2} = \frac{L}{2D}$ But $\alpha = \frac{2\lambda}{a_1}$ ; therefore, $L = \frac{2\lambda D}{a_1}$                                                                                                                                                                                                            |  | 0.75 |
|       | 1.3.3                                                                                                | $d = \frac{L}{2} = \frac{2 \times 600 \times 10^{-9} \times 2}{2 \times 0.1 \times 10^{-3}} = 0.012 \text{ m} = 12 \text{ mm}$                                                                                                                                                                                                                                                                                                                    | 0.5                                                                                 |      |
| 1.3.4 | $x_P < d = \frac{L}{2}$ , then it is neither the center of a bright nor the center of a dark fringe. | 0.25                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                     |      |
| 1.4   | Wave aspect of light                                                                                 | 0.25                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                     |      |
| 2     | 2.1                                                                                                  | $W_0$ is the minimum energy needed to extract an electron from the surface of a metal.                                                                                                                                                                                                                                                                                                                                                            | 0.5                                                                                 |      |
|       | 2.2                                                                                                  | $E_{ph} = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}} = 3.3 \times 10^{-19} \text{ J}$<br>$E_{ph} = \frac{3.3 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.0625 \text{ eV}$                                                                                                                                                                                                                             | 0.75                                                                                |      |
|       | 2.3                                                                                                  | $E_{ph} < W_0$ , then there is no photoelectric emission.                                                                                                                                                                                                                                                                                                                                                                                         | 0.25                                                                                |      |
|       | 2.4                                                                                                  | $E'_{ph} = W_0 + KE_{max}$ , then $KE_{max} = E'_{ph} - W_0 = \frac{hc}{\lambda'} - W_0$<br>$KE_{max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9} \times 1.6 \times 10^{-19}} - 2.39 = 0.085 \text{ eV}$                                                                                                                                                                                                                 | 0.75                                                                                |      |
|       | 2.5                                                                                                  | Corpuscular (particle) aspect of time                                                                                                                                                                                                                                                                                                                                                                                                             | 0.25                                                                                |      |