وزارة التربية والتعليم العالي
المديرية العامّة للتربية
دائرة الامتحانات الرسميّة

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

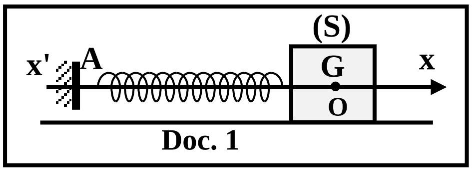
دورة العام ٢٠٢١ العاديّة
الاثنين ٢٦ تموز ٢٠٢١

مسابقة في مادة الفيزياء — الاسم:
المدة: ساعة ونصف — الرقم:

**This exam is formed of three obligatory exercises in three pages.**
**The use of non-programmable calculator is recommended.**

## Exercise 1 (7 pts) Mechanical oscillator

A mechanical oscillator is constituted of a block (S) of mass M and a spring of negligible mass and force constant k.
The spring, placed horizontally, is connected from one of its extremities to a fixed support A. (S) is attached to the other extremity of the spring and it may slide without friction on a horizontal surface (Doc. 1).



Doc. 1

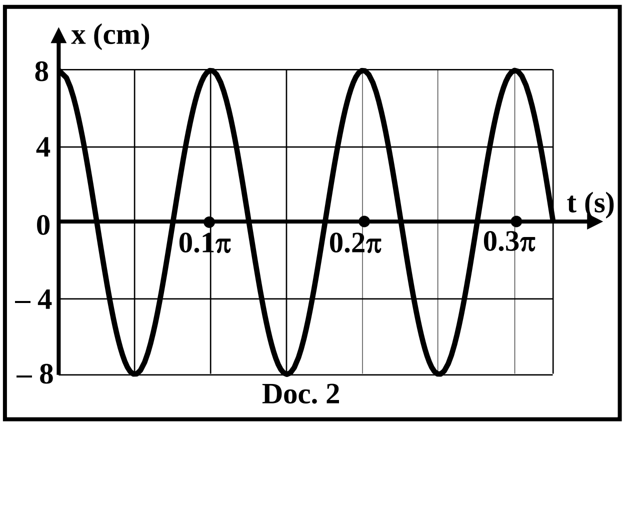
The aim of this exercise is to determine the values of M and k.
At equilibrium, the center of mass G of (S) coincides with the origin O of the x-axis.
(S) is shifted from its equilibrium position in the positive direction and then released without initial velocity at the instant $t_0 = 0$. Thus, (S) performs mechanical oscillations. At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.
The horizontal plane containing G is considered as a reference level for gravitational potential energy.

**1)** Write, at an instant t, the expression of the mechanical energy ME of the system (Oscillator, Earth) in terms of x, M, k and v.

**2)** Establish the second order differential equation in x that governs the motion of G.

**3)** Deduce that the expression of the proper (natural) period of the oscillations is $T_1 = 2\pi\sqrt{\frac{M}{k}}$.

**4)** An appropriate device traces x as a function of time (Doc. 2).
Referring to document 2, indicate:
**4.1)** the type of oscillations of G;
**4.2)** the amplitude $X_m$ of the oscillations;
**4.3)** the value of $T_1$.



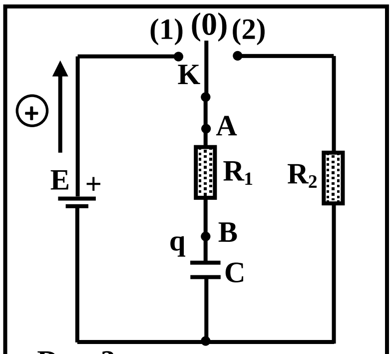
Doc. 2

**5)** The same experiment is repeated by putting on (S) an object, considered as a particle, of mass m = 50 g. The duration of 10 oscillations becomes $\Delta t = 3.67$ s.
**5.1)** Write the expression of the new proper (natural) period $T_2$ of the oscillations in terms of k, M and m.
**5.2)** Using the expressions of $T_1$ and $T_2$, show that $k = \frac{4\pi^2 m}{T_2^2 - T_1^2}$.
**5.3)** Determine the values of k and M.

## Exercise 2 (7 pts) Charging and discharging a capacitor

The aim of this exercise is to study the charging and the discharging of a capacitor.
For this purpose, we set up the circuit of document 3 that includes:

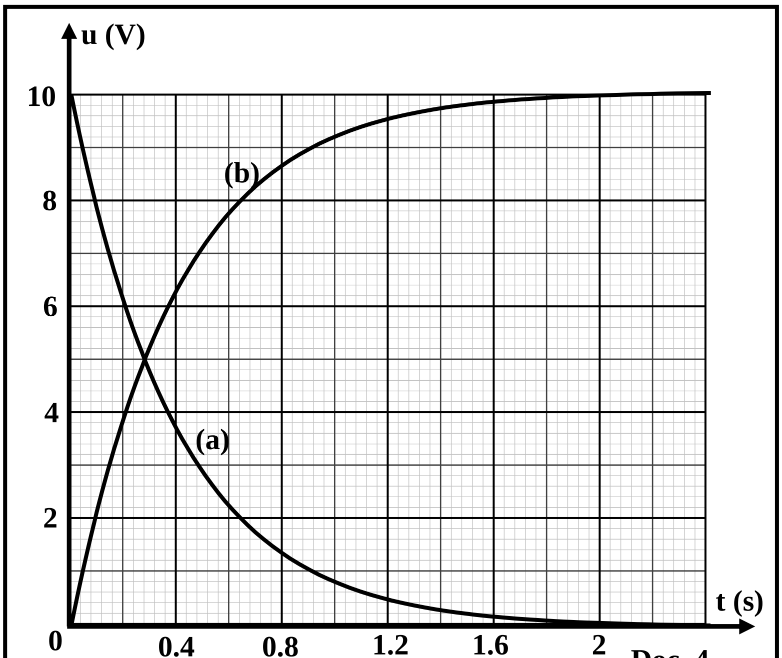
- an ideal battery of electromotive force E = 10 V;
- two resistors of resistances $R_1 = R_2 = 4\ k\Omega$;
- a capacitor of capacitance C;
- a switch K.



Doc. 3

**1) Charging the capacitor**

The switch K is initially at position (0) and the capacitor is uncharged.
At the instant $t_0 = 0$, K is turned to position (1) and the charging process of the capacitor starts.
At an instant t, plate B of the capacitor carries a charge q and the circuit carries a current i.
An appropriate device allows us to display the voltage $u_{AB} = u_{R_1}$ across the resistor and the voltage $u_{BD} = u_C$ across the capacitor.
Curves (a) and (b) of document 4 show these voltages as functions of time.



Doc. 4

**1.1)** Curve (a) represents $u_{R_1}$ and curve (b) represents $u_C$. Justify.

**1.2)** The time constant of this circuit is given by $\tau_1 = R_1 C$.

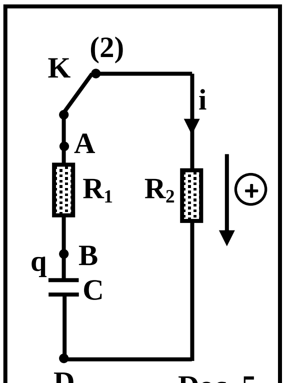
**1.2.1)** Using document 4, determine the value of $\tau_1$.

**1.2.2)** Deduce the value of C.

**1.3)** Calculate the time « $t_1$ » needed by the capacitor to practically become completely charged.

**2) Discharging the capacitor**

The capacitor is completely charged. At an instant taken as a new initial time $t_0 = 0$, the switch K is turned to position (2), and the capacitor starts discharging through the resistors of resistances $R_1$ and $R_2$. At an instant t the circuit carries a current i (Doc. 5).



Doc. 5

**2.1)** Show, using the law of addition of voltages, that the differential equation which governs $u_C$ is:

$$RC\frac{du_C}{dt} + u_C = 0 \text{ where } R = R_1 + R_2.$$

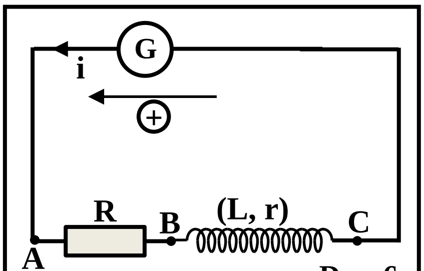
**2.2)** The solution of this differential equation is of the form: $u_C = E\, e^{\frac{-t}{\tau_2}}$ where $\tau_2$ is the time constant of the circuit of document 5.
Determine the expression of $\tau_2$ in terms of R and C.

**2.3)** Verify that the time needed by the capacitor to practically become completely discharged is $t_2 = 5\tau_2$.

**3) Duration of charging and discharging the capacitor**

Show, without calculation, that « $t_2$ » is greater than « $t_1$ ».

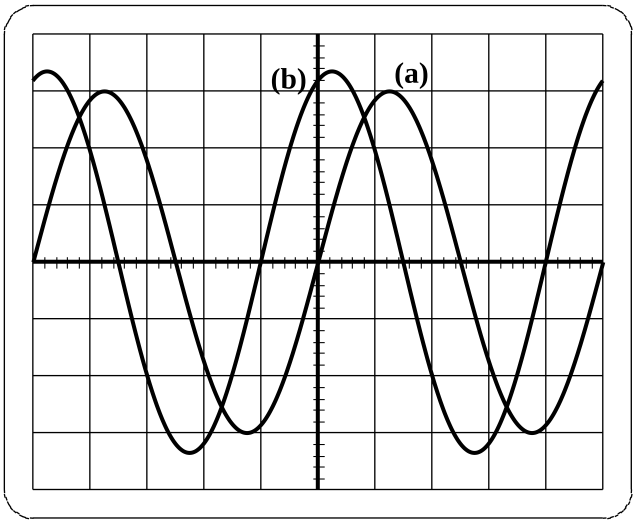
## Exercise 3 (6 pts) Characteristics of a coil



Doc. 6

In order to determine the inductance L and the resistance r of a coil, we connect it in series with a resistor of resistance R = 30 Ω across a function generator (G) providing an alternating sinusoidal voltage of angular frequency ω.
The circuit thus carries an alternating sinusoidal current of expression $i = I_m \sin(\omega t)$ (Doc. 6).
An oscilloscope allows us to display the voltage $u_{AB} = u_R$ across the resistor and the voltage $u_{BC} = u_L$ across the coil.
The obtained waveforms are shown in document 7.
The adjustments of the oscilloscope are:

- vertical sensitivity for both channels: $S_v$ = 2 V/div;
- horizontal sensitivity: $S_h$ = 0.4 ms/div.



Doc.7

**1)** The voltage $u_R$ represents the image of i. Why?
**2)** Referring to document 7, specify which of the curves, (a) or (b), leads the other.
**3)** Deduce that curve (a) corresponds to $u_{AB}$.
**4)** Using document 7, determine:
  **4.1)** the angular frequency ω;
  **4.2)** the maximum value $I_m$ of i ;
  **4.3)** the phase difference φ between $u_L$ and i.
**5)** Prove that $u_L = 6.8 \sin(\omega t + 0.4\pi)$ (SI).
**6)** Knowing that the voltage across the coil is given by $u_L = r\,i + L\frac{di}{dt}$, write the expression of $u_L$ in terms of r, L, ω and t.
**7)** Using the two expressions of $u_L$ found in parts 5 and 6 and by giving «ωt» two particular values, determine the values of L and r.

امتحانات الشهادة الثانوية العامة
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دورة العام 2021 العاديّة
الاثنين 26 تموز 2021

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|---|---|
| مسابقة في مادة الفيزياء | الاسم: |
| المدة: ساعة ونصف | الرقم: |

**Exercise 1 (7 pts)** **Mechanical oscillator**

| Part | | Answer | Note |
|---|---|---|---|
| 1 | | $ME = KE + EPE = \frac{1}{2} M v^2 + \frac{1}{2} kx^2$ | 0.5 |
| 2 | | The sum of the works done by the non-conservative forces is zero, then the mechanical energy is conserved. (Or: Friction is neglected, then the mechanical energy is conserved).<br>ME = constant , then $\frac{dME}{dt} = 0$ , so $M v v' + k x x' = 0$ , but $v = x'$ and $v' = x''$ , hence $v (M x'' + k x) = 0$<br>$v = 0$ is rejected , then $x'' + \frac{k}{M} x = 0$ | 1 |
| 3 | | The differential equation is of the form: $x'' + \omega_0^2 x = 0$ , with $\omega_o = \sqrt{\frac{k}{M}}$<br>$T_1 = \frac{2\pi}{\omega_o}$ ; therefore, $T_1 = 2\pi\sqrt{\frac{M}{k}}$ | 1 |
| 4 | 4.1 | Free undamped mechanical oscillations | 0.5 |
| | 4.2 | $X_m = 8$ cm | 0.5 |
| | 4.3 | From the curve: $T_1 = 0.1\,\pi$ s = 0.314 s | 0.5 |
| 5 | 5.1 | $T_2 = 2\pi\sqrt{\frac{M+m}{k}}$ | 0.5 |
| | 5.2 | $T_1^2 = 4\pi^2 \frac{M}{k}$ and $T_2^2 = 4\pi^2 (\frac{M+m}{k})$<br>$T_2^2 - T_1^2 = 4\pi^2 (\frac{M+m}{k} - \frac{M}{k}) = \frac{4\pi^2 m}{k}$ , so $k = \frac{4\pi^2 m}{(T_2^2 - T_1^2)}$ | 1 |
| | 5.3 | $T_2 = \frac{3.67}{10} = 0.367$ s<br>$k = \frac{4\pi^2 \times 0.05}{0.367^2 - 0.314^2}$ , then $k = 54.7$ N/m<br>$T_1^2 = 4\pi^2 \frac{M}{k}$ , substituting the value of k into this expression gives:<br>$0.314^2 = 4\pi^2 \frac{M}{54.7}$ ; therefore, $M = 0.1366$ kg = 136.6 g | 0.5<br>0.5<br>0.5 |

**Exercise 2 (7 pts)** **Charging and discharging of a capacitor**

| Part | | Answer | Note |
|---|---|---|---|
| 1 | 1.1 | Curve (a): $u_{AB} = u_{R_1} = R_1 i$ ; $u_{R_1}$ is directly proportional to the current in the circuit.<br>During the charging process, the current decreases so $u_{R_1}$ decreases.<br>Curve (b) : $u_{BD} = u_C = \frac{q}{C}$ ; During charging process q increases so $u_C$ increases | **0.5**<br>**0.5** |
| | 1.2.1 | At $t = \tau_1$ : $u_C = 0.63\ E = 6.3$ V<br>From document 4: $u_C = 6.3$ V at $t = 0.4$ s , then $\tau_1 = 0.4$ s | **1** |
| | 1.2.2 | $\tau_1 = R_1 C$ , so $C = \frac{\tau_1}{R_1} = \frac{0.4}{4000}$ , hence $C = 1 \times 10^{-4}$ F = 100 µF | **0.5** |
| | 1.3 | $t_1 = 5\tau_1 = 5 \times 0.4$ , then $t_1 = 2$ s | **0.5** |
| 2 | 2.1 | $u_{BD} = u_{BA} + u_{AD}$<br>$u_C = R_1 i + R_2 i$ , then $u_C = (R_2 + R_1)\, i = R\, i$<br>But, $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ , hence $u_C = -RC\frac{du_C}{dt}$<br>Therefore , $RC\frac{du_C}{dt} + u_C = 0$ | **1.5** |
| | 2.2 | $u_C = E\, e^{\frac{-t}{\tau_2}}$ , then $\frac{du_C}{dt} = -\frac{E}{\tau_2} e^{\frac{-t}{\tau_2}}$<br>Substituting $u_C$ and $\frac{du_C}{dt}$ into the differential equation gives:<br>$RC(-\frac{E}{\tau_2} e^{\frac{-t}{\tau_2}}) + E\, e^{\frac{-t}{\tau_2}} = 0$ , so $E\, e^{\frac{-t}{\tau_2}}(1 - \frac{RC}{\tau_2}) = 0$<br>$E\, e^{\frac{-t}{\tau_2}} = 0$ is rejected , then $1 - \frac{RC}{\tau_2} = 0$ , so $\tau_2 = RC$ | **1.5** |
| | 2.3 | At $t = 5\,\tau_2$ : $u_C = E\, e^{\frac{-5\tau_2}{\tau_2}} = E\, e^{-5} \cong 0$ , so the capacitor is practically completely. discharged. | **0.5** |
| 3 | | $t_1 = 5 R_1 C$ and $t_2 = 5RC = 5(R_1 + R_2)C$<br>$(R_1 + R_2) > R_1$ , then $t_2 > t_1$ | **0.5** |

**Exercise 3 (6 pts)** **Characteristics of a coil**

| Part | | Answer | Note |
|---|---|---|---|
| 1 | | $u_R = Ri$ , but R is a positive constant , then $u_R$ and i are directly proportional ; therefore, $u_R$ is the image of current. | **0.5** |
| 2 | | Curve (b) leads curve (a), since curve (b) becomes maximum before curve (a). | **0.5** |
| 3 | | The voltage across the coil $u_L$ leads $u_R$ (or i).<br>Curve (b) leads curve (a), then curve (a) corresponds to $u_R = u_{AB}$. | **0.5** |
| 4 | 4.1 | $T = 5 \times 0.4 = 2$ ms $= 2\times10^{-3}$ s<br>$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\times10^{-3}}$ , hence $\omega = 1000\,\pi$ rad/s | **0.25**<br>**0.5** |
| | 4.2 | Curve (a): $U_{R(max)} = 3 \times 2 = 6$ V<br>$U_{R(max)} = R \times I_m$ , then $I_m = \frac{6}{30} = 0.2$ A | **0.25**<br>**0.5** |
| | 4.3 | $\varphi = \frac{2\pi d}{D} = \frac{2\pi\times1}{5}$ , then $\varphi = 0.4\,\pi$ rad | **0.5** |
| 5 | | From curve (b): $U_{L(max)} = 3.4 \times 2 = 6.8$ V , and $u_L$ leads i by $\varphi = 0.4\pi$ rad<br>$u_L = U_{L(max)} \sin(\omega t + \varphi)$ ; therefore, $u_L = 6.8 \sin(\omega t + 0.4\,\pi)$ | **0.25**<br>**0.25** |
| 6 | | $u_L = r\,i + L\frac{di}{dt} = r\,I_m \sin(\omega t) + L\,I_m\,\omega \cos(\omega t)$<br>$u_L = 0.2\,r \sin(\omega t) + L\,(0.2)\,(1000\pi) \cos(\omega t) = 0.2\,r \sin(\omega t) + 200\pi\,L \cos(\omega t)$ (SI)<br>**<u>Or</u>** $u_L = 0.2\,r \sin(\omega t) + \omega\,L\,(0.2) \cos(\omega t)$ (SI) | **0.5** |
| 7 | | $6.8 \sin(\omega t + 0.4\,\pi) = 0.2\,r \sin(\omega t) + 200\,\pi\,L \cos(\omega t)$<br>For $\omega t = 0$ : $6.8 \sin(0.4\,\pi) = 0 + 200\pi\,L$ , then $L = 0.01$ H<br>For $\omega t = \frac{\pi}{2}$ rad : $6.8 \sin(\frac{\pi}{2} + 0.4\,\pi) = 0.2\,r + 0$ , then $r = 10.5\ \Omega$ | **0.75**<br>**0.75** |